

Overall, informal outline

- SMBH mass measurements and the various techniques → why do we care about mass measurements in the first place?
 - Help inform assembly history and timeline of galaxy formation, understand star formation through feedback with the central black hole and quenching, understand current population of SMBHs and their demographics, understand and predict the gravitational wave signal from binary SMBHs, etc
 - Trace build up of mass in SMBHs over cosmic time/growth channels of galaxies over time
 - These are tough questions - MASSIVE survey was designed to mitigate these challenges and understand massive galaxy formation and the supermassive black holes at the centers of these large ellipticals
- Data Processing – what data do we use? How do we go from photometry and spectra to inputs?
 - Need to mention: exquisite IFU and photometry needed; IFU data used for full spectrum fitting for the velocity profiles of these galaxies expressed as Gauss-Hermite moments
 - Photometry needed to model the mass distribution of the stars
 - Show some typical velocity profiles, and maybe some unique ones that make the modeling challenging
- Modeling
 - Intro to axisymmetric and triaxial Schwarzschild models; TriOS code; give a sense of how complex these models are
 - Superposition of orbits is constructed to mimic the input light profile and kinematics from densely sampled orbits in phase space
- Turning galaxy models into parameters
 - Briefly run through nested sampling routine and how we extract posterior information on these parameters
 - Maybe introduce here that for axisymmetric models, more often than not our models prefer edge-on inclinations and this was a sign that maybe these models are too flexible for their own good, leading to potential biases in the parameters
- Model Flexibility and Mock Recovery
 - We've been testing this idea at Berkeley on the more complicated triaxial models.
 - Axisymmetric: does seem that edge-on is preferred when the models are good enough, the covariance with the other parameters doesn't seem to be too strong
 - Triaxial: model flexibility is a less strong and less obvious effect, and in our tests, we don't seem to see a need to penalize our models by their intrinsic flexibilities; our models seem to do a decent job at recovering the inputs, especially at the level our measurement uncertainties limit us to
- Where we are now and what's to come

Stellar Dynamical Mass Measurements of MASSIVE Elliptical Galaxies

More details in:
Liepold+20,
Quenneville+21,+22,
Pilawa+22,
Pilawa+23 (coming soon!)

Jacob Pilawa (UCB),

February 24, 2023 @ TAMU

Emily Liepold (UCB), Matthew Quenneville (UCB), Chung-Pei Ma (UCB)

+ Silvana Delgado Andrade (TAMU), Jonelle Walsh (TAMU)

+ Jenny Greene (Princeton), John Blakeslee (NOIRLab)

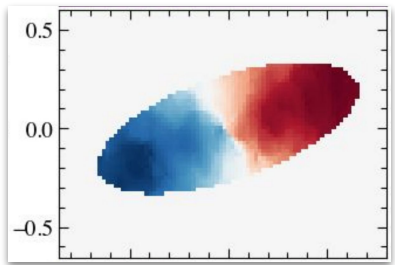
A roadmap for the talk:



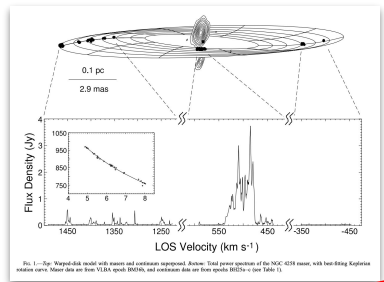
- 1. Supermassive black holes in the largest galaxies**
2. Triaxial Schwarzschild modeling of massive ellipticals
3. Robustness and Recovery Tests of Dynamical Modeling Techniques

main techniques for $z \sim 0$ “direct” BH discovery:

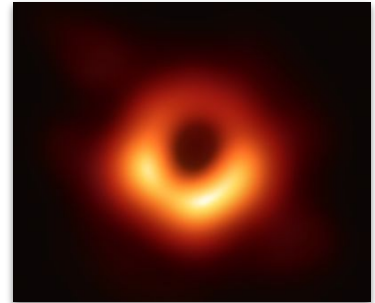
gas disk
dynamics



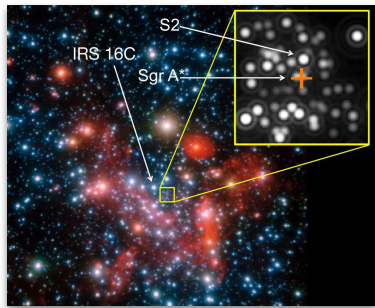
maser disks



EHT



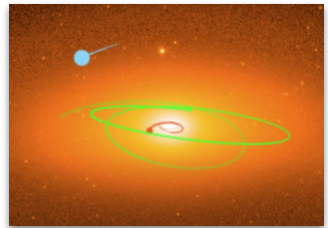
galactic
center



integrated
light



stellar
dynamics

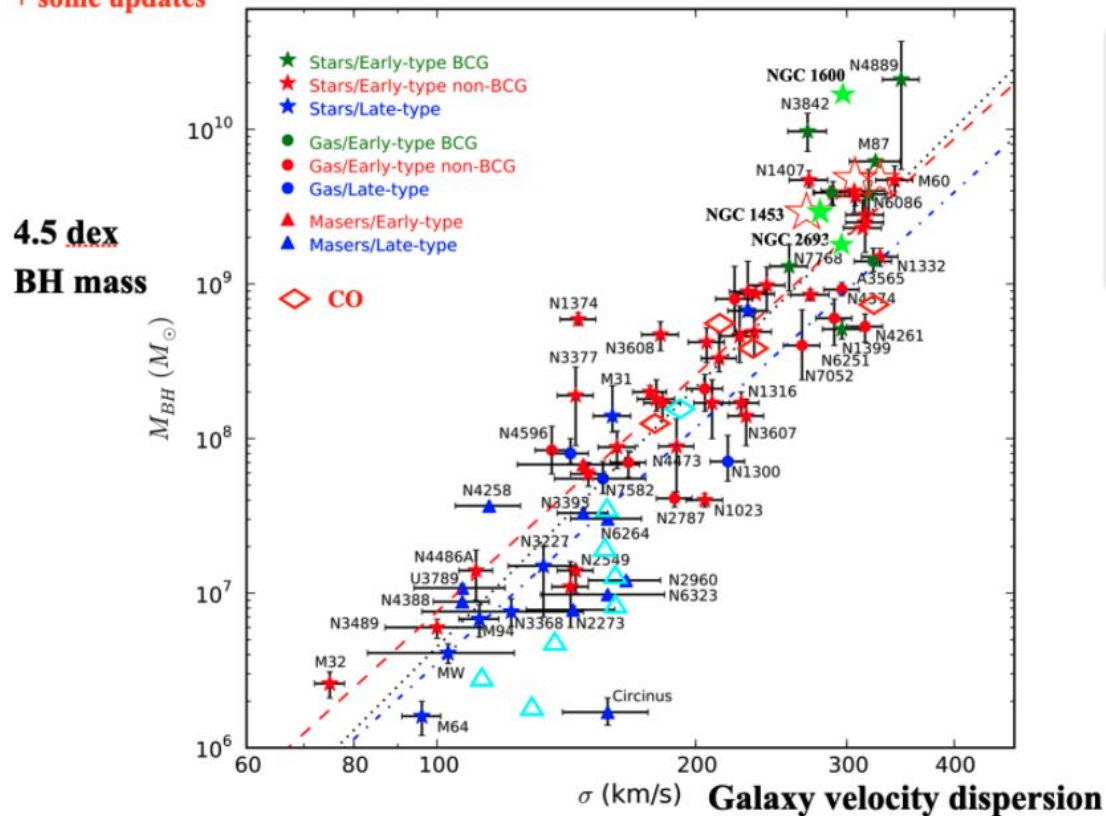


...

see: van den Bosch+08 (stellar),
Barth+01, Walsh+10 (gas dynamics),
Miyoshi+95, Herrnstein+99 (masers).
slide inspiration: J. Walsh (TAMU)

BLACK HOLES + GALAXY EVOLUTION

McConnell & Ma (2013)
+ some updates



$$r_{\text{SOI}} \approx \frac{GM}{\sigma^2} \approx 10\text{'s to } 100\text{'s of pc}$$

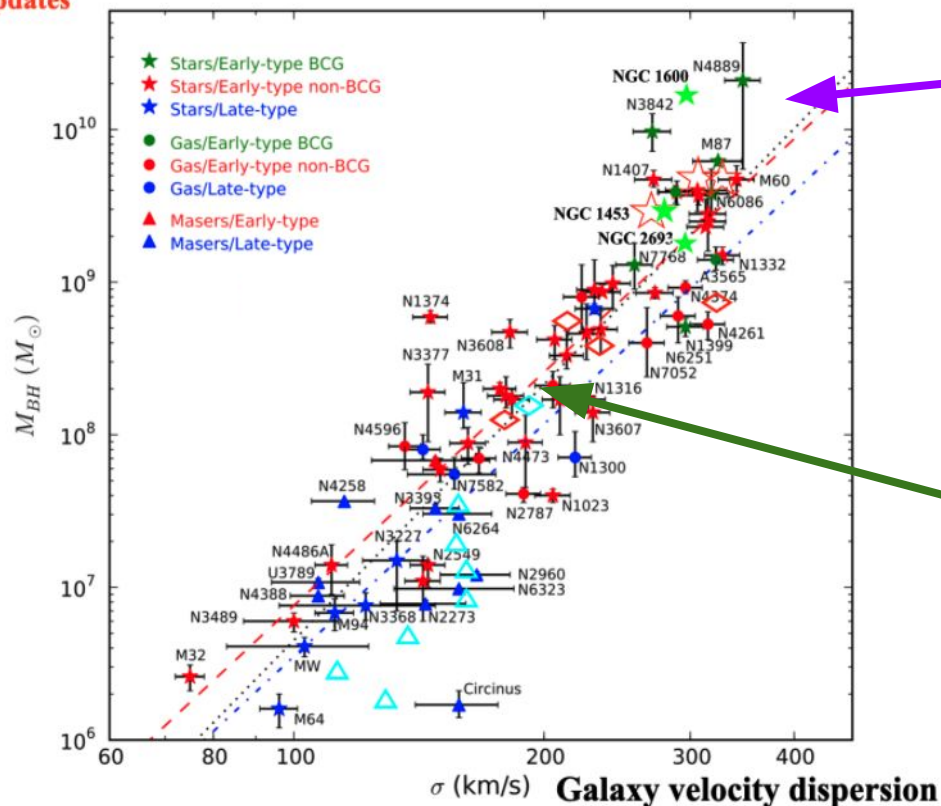
$$r_{\text{galaxy}} \approx 10\text{'s of kpc}$$

***how do galaxies
and SMBHs evolve
together?***

A CLUE: the most MASSIVE galaxies

McConnell & Ma (2013)

+ some updates



growth by gas-poor
mergers

- BH increases
- σ saturates
- kinematically misaligned,
slow rotation ($V/\sigma \sim < 0.2$)
- **indicative of triaxiality?**

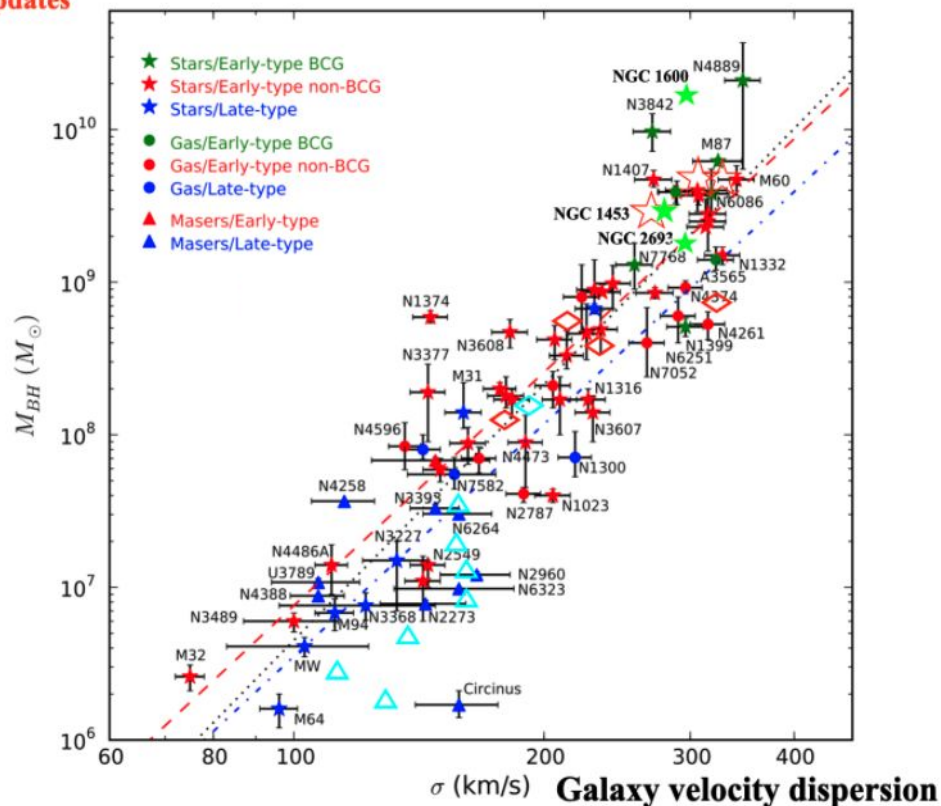
growth by gas
accretion/gas-rich mergers

- BH increases
- fast rotators ($V/\sigma > \sim 0.3$)
- kinematically aligned
- **axisymmetric shapes**

A CLUE: the most MASSIVE galaxies

McConnell & Ma (2013)

+ some updates



*our interpretation depends
on M_{BH} 's at the most
massive end*

1. Estimating BH Masses
2. Cross-checking of methods (e.g., ALMA gas dynamics [e.g., Cohn+2021, Kabasares+2022])
3. Calibration of reverberation mapped AGN
4. $z \sim 0$ mass function \rightarrow BBH merger rates
5. Comparison to simulations of AGN feedback modes and mechanisms

... but it's challenging...

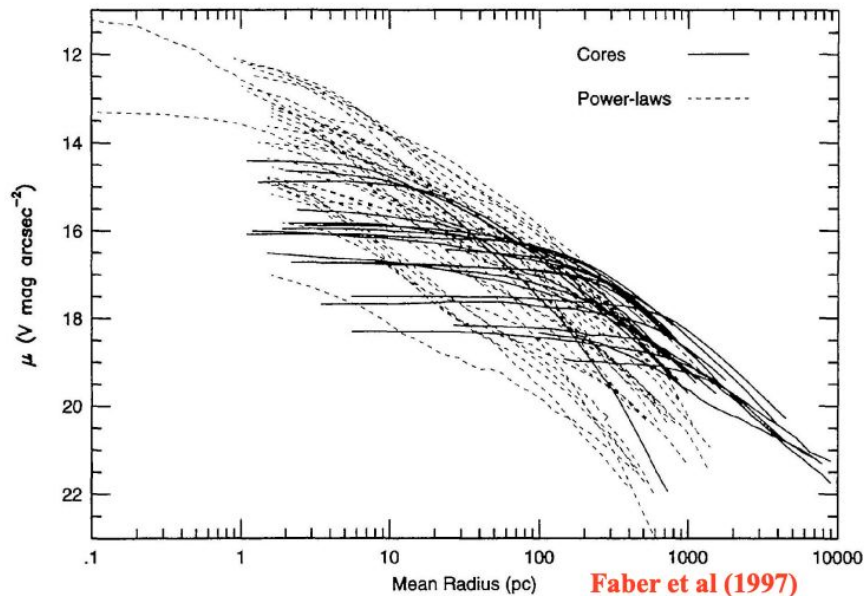
[i.e., Yu+2019 (reverberation mapping), Thater+21 (gas dynamics)],

[i.e., Shannon+2015, Arzoumanian+2019], [i.e., Li+2019, Habouzit+2020]

why are massive ellipticals a challenge?

they're both rare, and have
extremely faint/flat cores

sphere of influence
is **tiny**



$$r = \frac{GM_{BH}}{\sigma^2} \approx 50\text{pc} \frac{M_{BH}}{10^9 M_{\odot}} \left(\frac{300 \text{ km s}^{-1}}{\sigma} \right)^2$$

a few 0.1's arcsec at ~100Mpc

→ long exposures on 8-10m
telescopes!

The MASSIVE Survey

McDonald Observatory

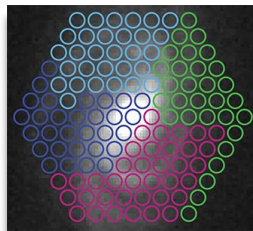


Gemini North



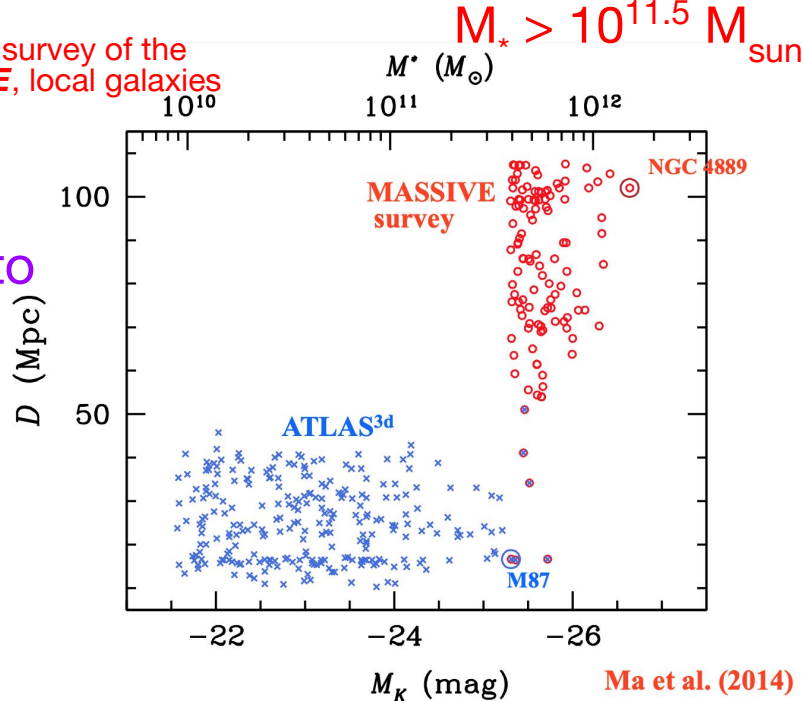
Photometry: CFHT, HST, UKIRT, PanSTARRS

Wide-field IFU
($107'' \times 107''$, out to
 $\sim 2 R_{\text{eff}}$)

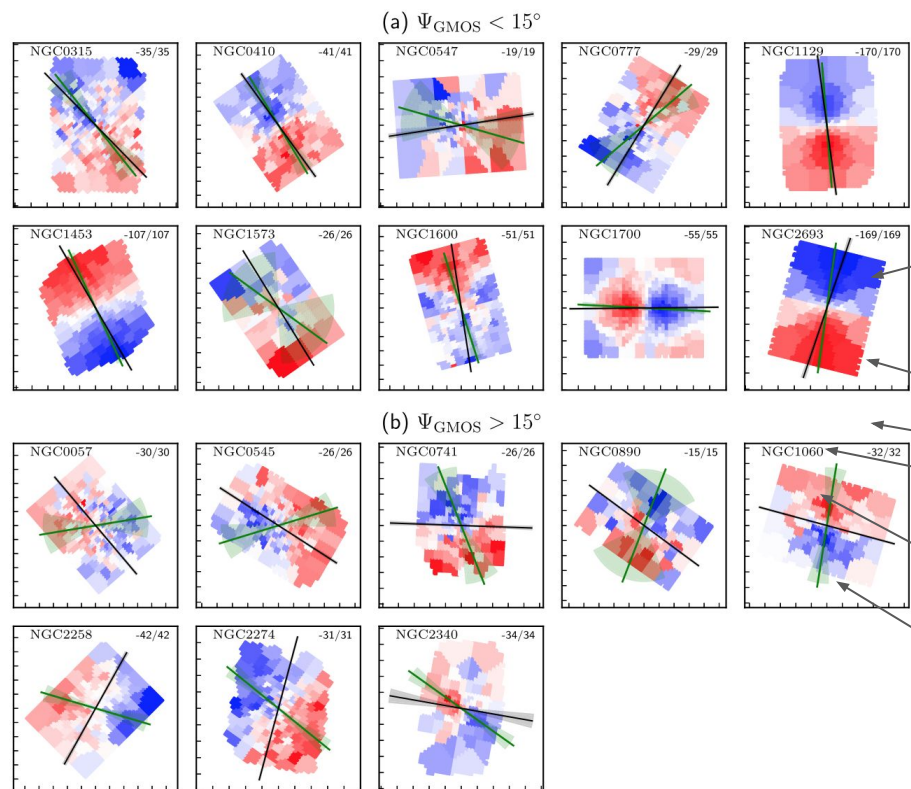


High-resolution,
high SNR IFU
($\sim 0.3''$ to $\sim 5''$ at
SNR ~ 125)

a **volume-limited** survey of the
 ~ 100 most **MASSIVE**, local galaxies



The MASSIVE Survey: Stellar Kinematics

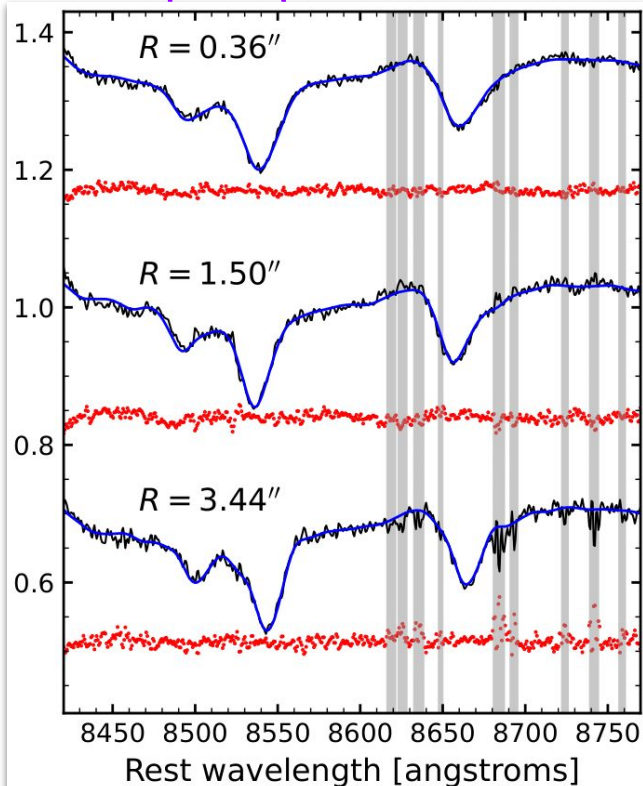


MASSIVE Survey Related Papers

I	Survey paper	Ma +	(2014) ApJ
II	Stellar pop gradient	Greene +	(2015) ApJ
III	Molecular gas #1	Davis +	(2016) MNRAS
IV	X-ray properties	Goulding +	(2016) ApJ
	NGC1600 black hole	Thomas +	(2016) Nature
V	Stellar kinematics	Veale +	(2017) MNRAS
VI	Ionized gas	Pandya +	(2017) ApJ
VII	λ & environment	Veale +	(2017) MNRAS
VIII	σ radial profile	Veale +	(2018) MNRAS
IX	WFC3 photometry	Goullaud +	(2018) ApJ
X	Kinematic alignment	Ene +	(2018) MNRAS
XI	Molecular gas #2	Davis +	(2019) MNRAS
XII	Stellar pop vs env	Greene +	(2019) ApJ
XIII	Core kinematics #1	Ene +	(2019) ApJ
XIV	Core kinematics #2	Ene +	(2020) ApJ
XV	NGC1453 black hole	Liebold +	(2020) ApJ
	SBF distances	Jensen +	(2021) ApJ
	SBF H_0	Blakeslee +	(2021) ApJ
	Axisymmetric orbit code	Quenneville +	(2021) ApJ
	Triaxial orbit code	Quenneville +	(2022) ApJ
XVI	Stellar pop & IMF	Gu +	(2022) ApJ
XVII	NGC2693 black hole	Pilawa +	(2022) ApJ
XVIII	CFHT imaging	Quenneville +	(2022) submitted
	Globular clusters	Hartmann +	(2023) submitted

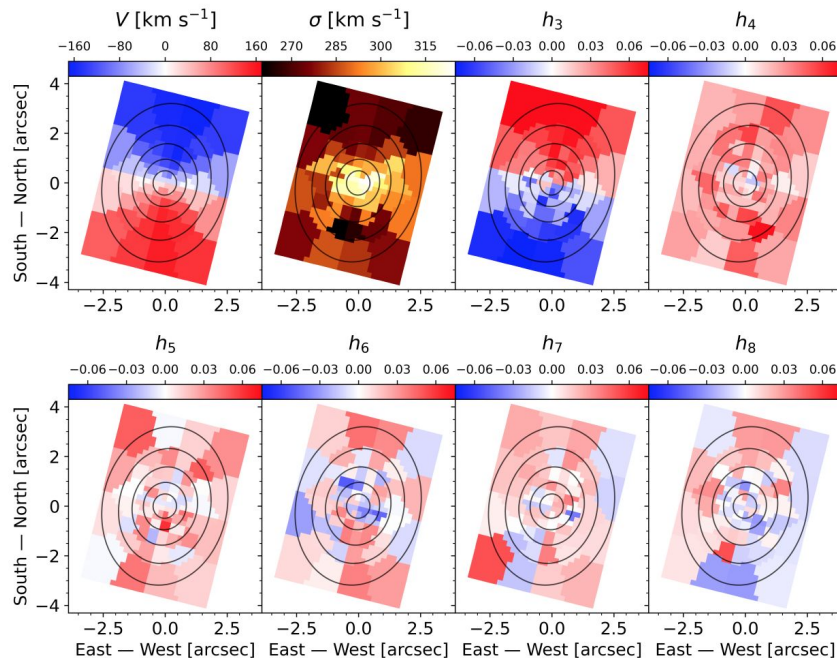
The MASSIVE Survey: Stellar Kinematics

Observables:
3 sample spectra from GMOS



measure **spatially resolved** line-of-sight
velocity distributions of stars

$$f(v) \propto \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(v-V)^2}{\sigma^2}} \left[1 + \sum_{m=3}^n h_m H_m \left(\frac{v-V}{\sigma} \right) \right]$$



A roadmap for the talk:

1. Supermassive black holes in the largest galaxies



2. **Triaxial Schwarzschild modeling of massive ellipticals**

3. Robustness and Recovery Tests of Dynamical Modeling Techniques

dynamical modeling

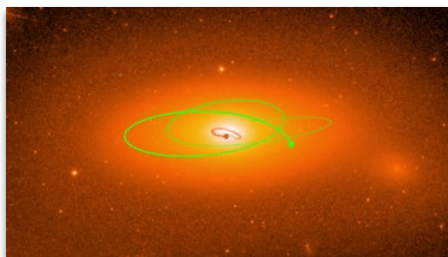
axisymmetric systems

a new code for
Schwarzschild modeling:

TriOS: Triaxial Orbit Superposition

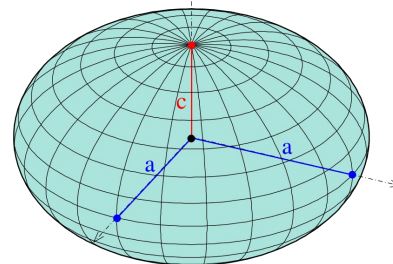
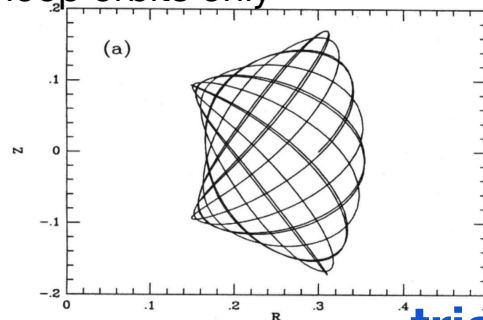
(van den Bosch+08,
Quenneville+21 updates)

inputs: stellar kinematics, surface
brightness; galaxy model parameters

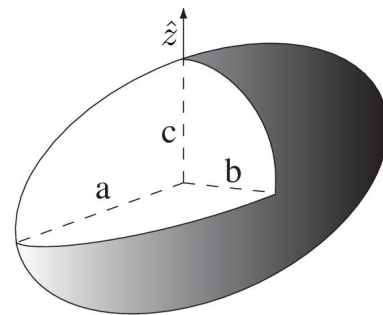
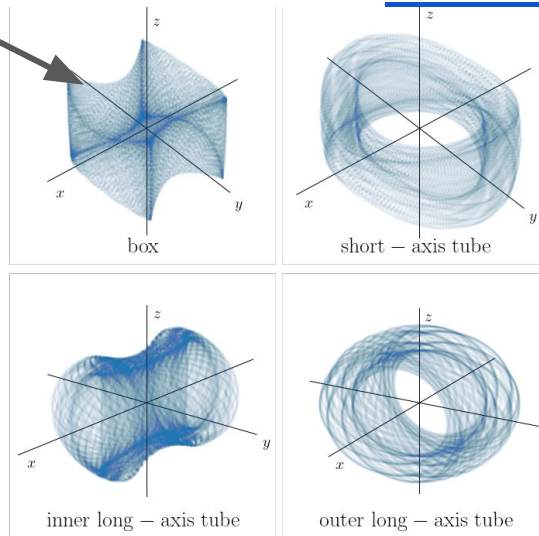


outputs: set of stellar orbits which best
reproduces the input kinematics

loop orbits only



triaxial systems



dynamical modeling

1. Choose a trial potential:

$$\text{Galaxy} = (\text{DM}) + (\text{STARS}) + (\text{BH}) + \dots$$

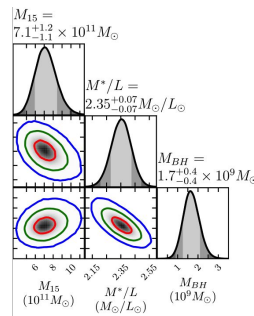
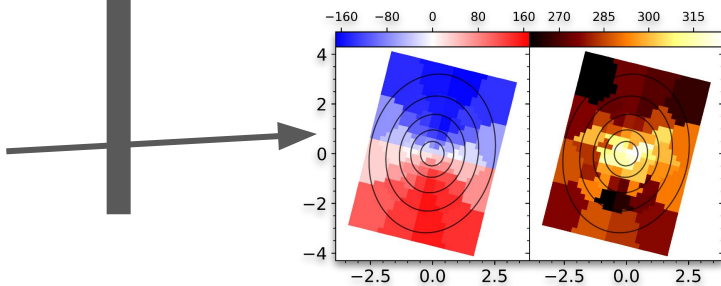
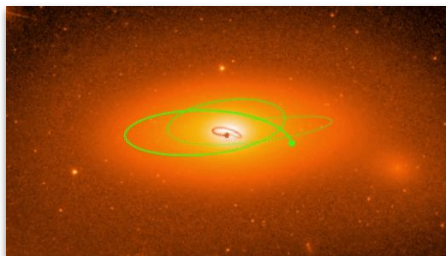
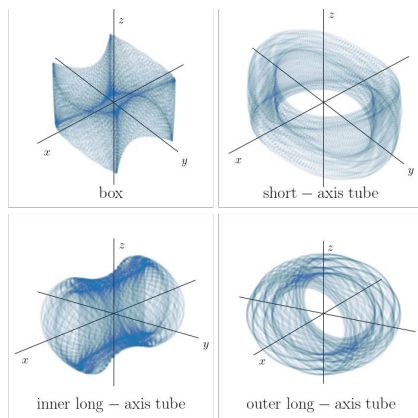
2. Generate stellar orbits in trial potential

3. Determine which orbits most accurately reproduce **kinematics** + **photometry** for a *single* trial potential

4. Find which assumed potential fits **kinematics** + **photometry** best **across** trial potentials (BH, ML, Shapes)

triaxial: BH, M/L, DM Halo, 3 shape parameters

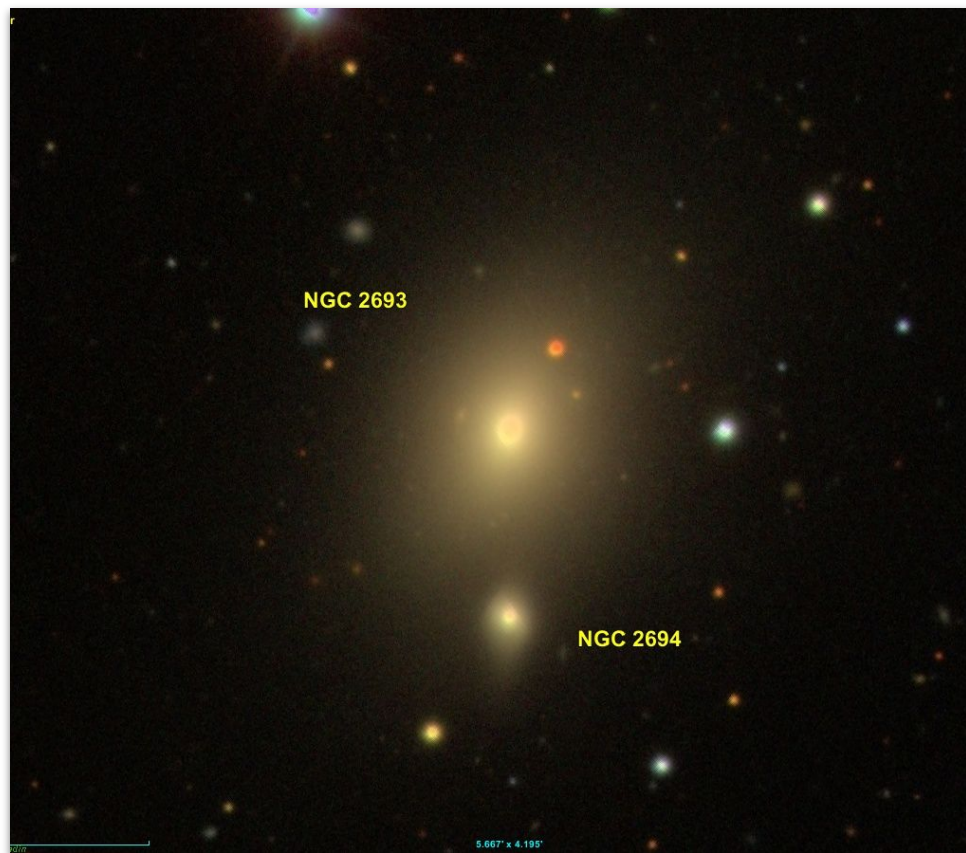
axisymmetric: BH, M/L, DM Halo, inclination



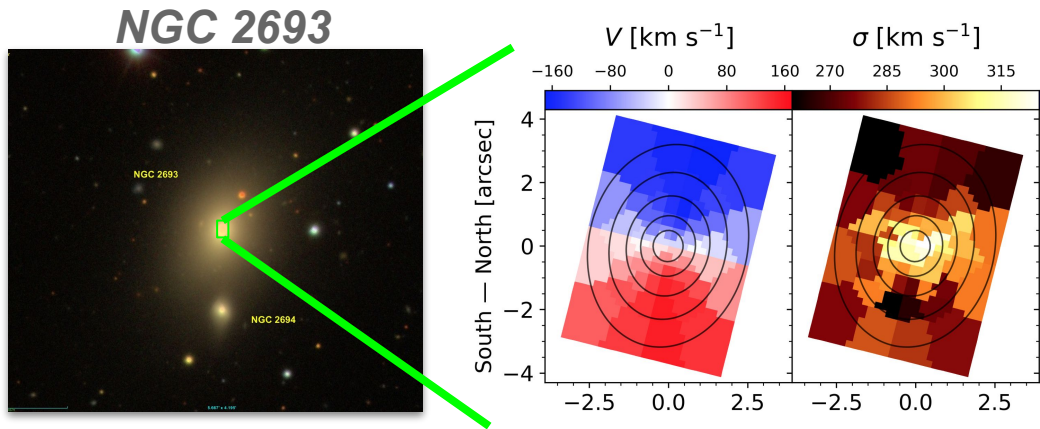
stellar kinematics of NGC 2693

Liepold+20
Quenneville+21
Pilawa+22

NGC 2693



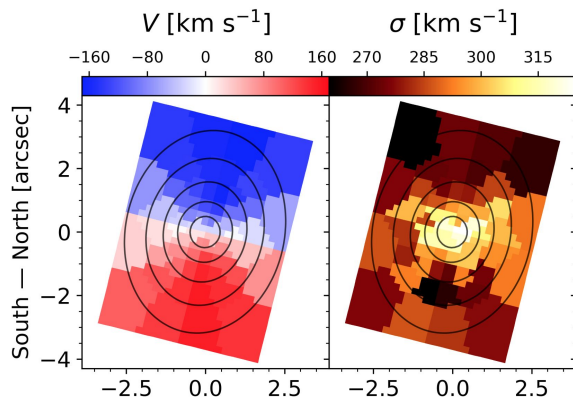
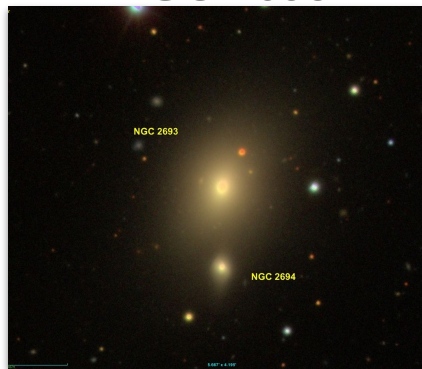
stellar kinematics of NGC 2693



- regular, fast rotator
($V \sim 150$ km/s,
 $\sigma \sim 320$ km/s)

stellar kinematics of NGC 2693

NGC 2693

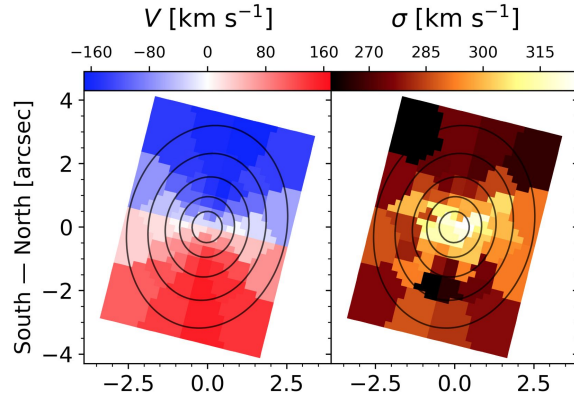
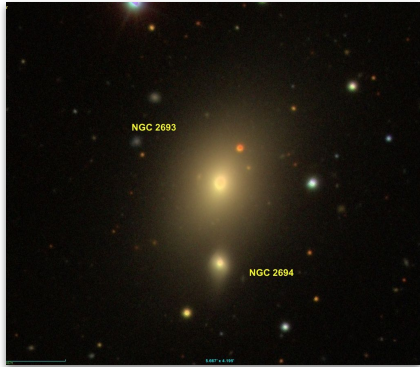


- regular, fast rotator ($V \sim 150 \text{ km/s}$, $\sigma \sim 320 \text{ km/s}$)
- mostly regular, elliptical isophotes

stellar kinematics of NGC 2693

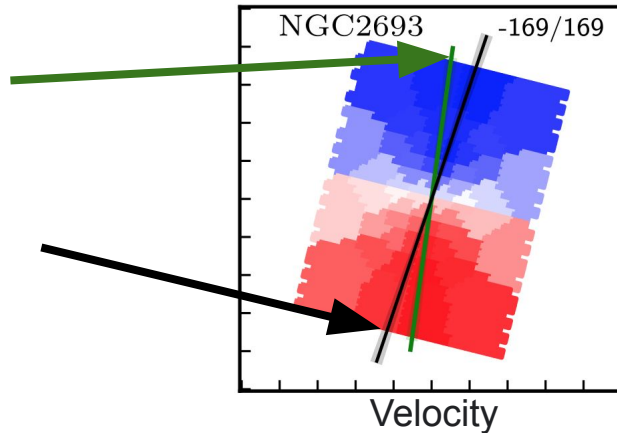
Liepold+20
Quenneville+21
Pilawa+22

NGC 2693



average
orientation of
stellar motion

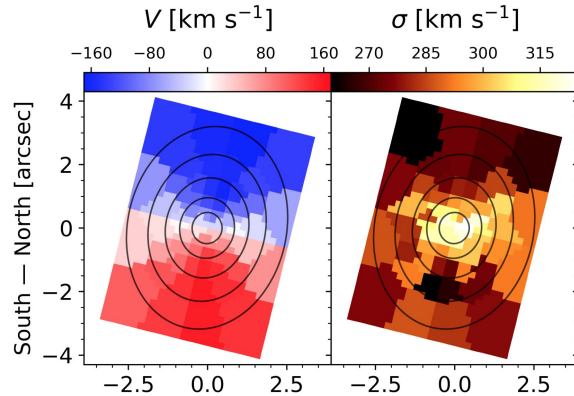
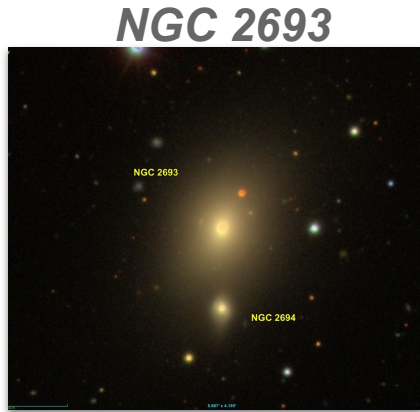
photometric
major axis



- regular, fast rotator ($V \sim 150 \text{ km/s}$, $\sigma \sim 320 \text{ km/s}$)
- mostly regular, elliptical isophotes
- **kinematic** and **photometric** major axes are *nearly* aligned ($\Delta\Psi \sim 5^\circ$)

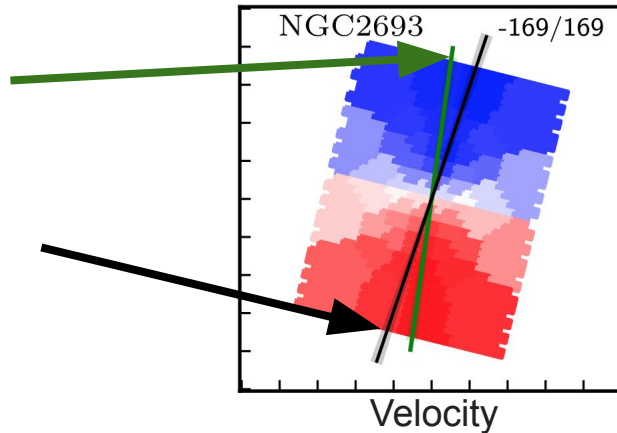
stellar kinematics of NGC 2693

Liepold+20
Quenneville+21
Pilawa+22



average
orientation of
stellar motion

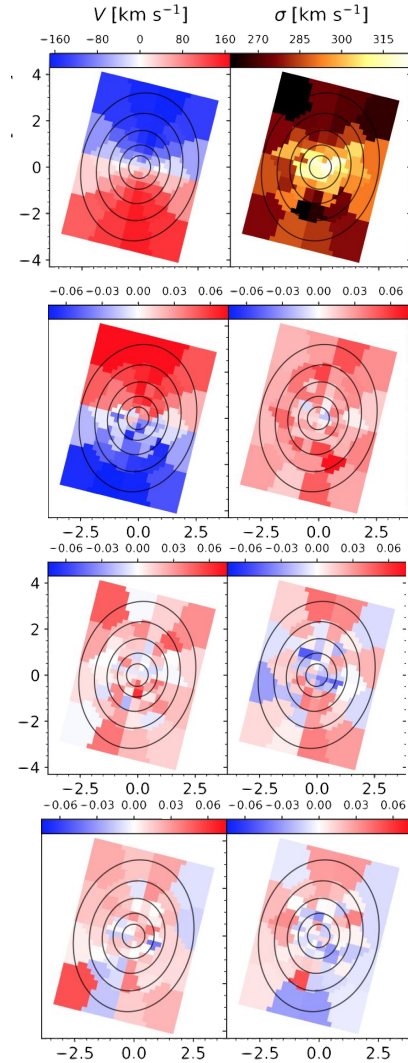
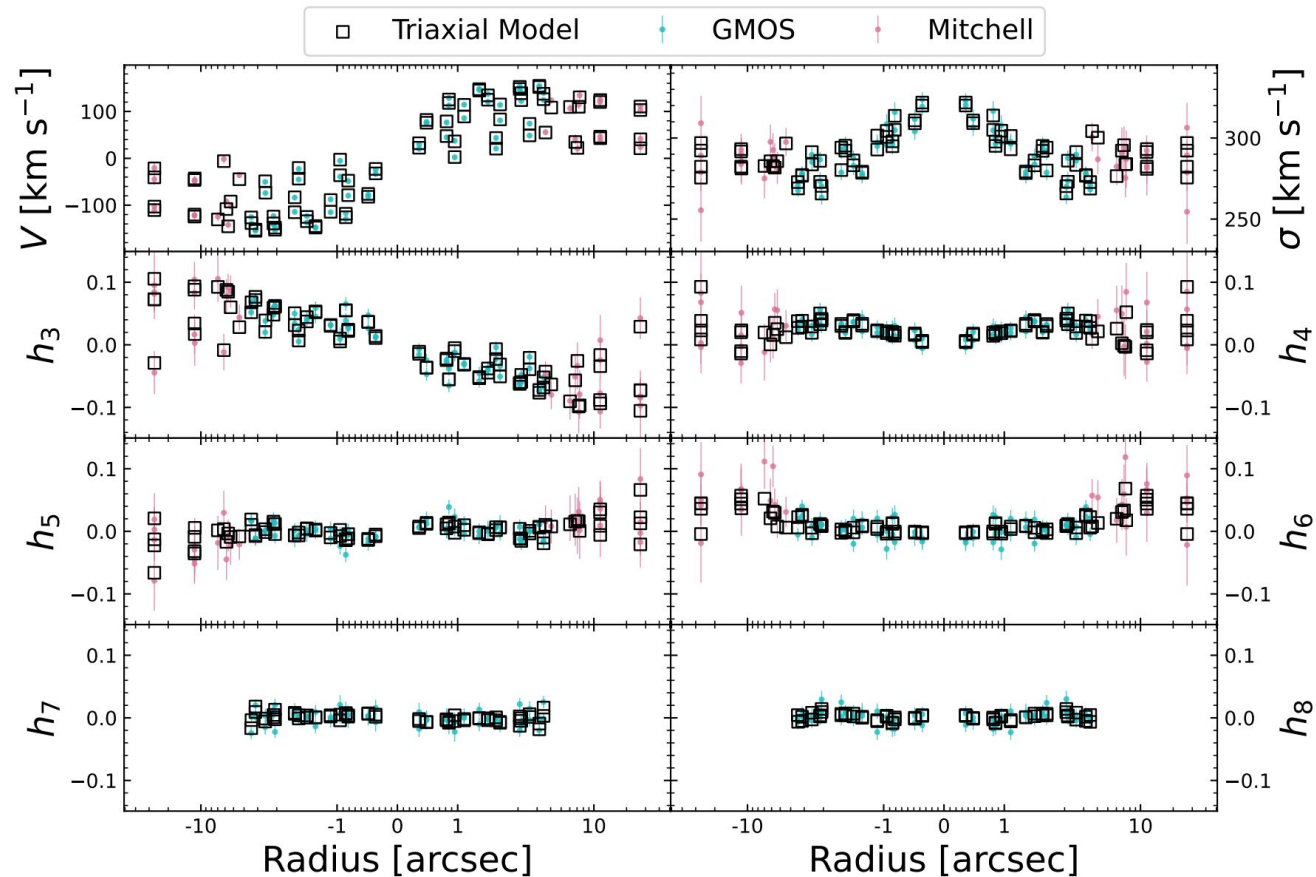
photometric
major axis



- regular, fast rotator ($V \sim 150$ km/s, $\sigma \sim 320$ km/s)
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- **kinematic** and **photometric** major axes are *nearly* aligned ($\Delta\Psi \sim 5^\circ$)

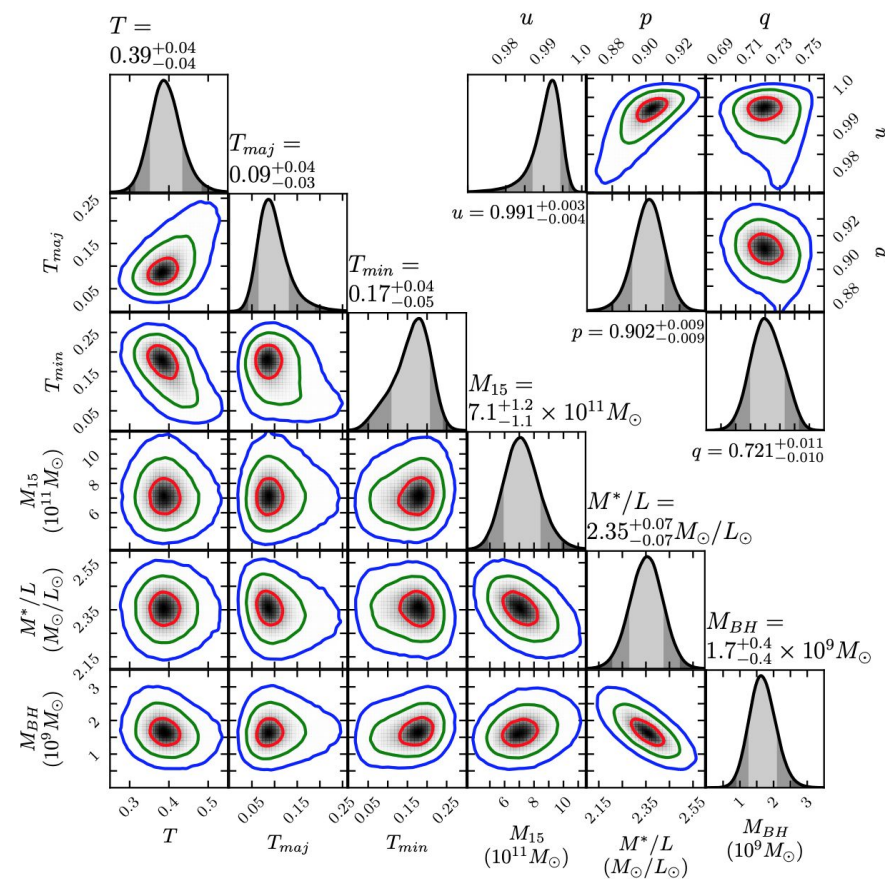
properties *nearly* consistent with **axisymmetric** intrinsic shapes, so let's test!

stellar dynamical modeling: applications to NGC 2693



stellar dynamical modeling: applications to NGC 2693

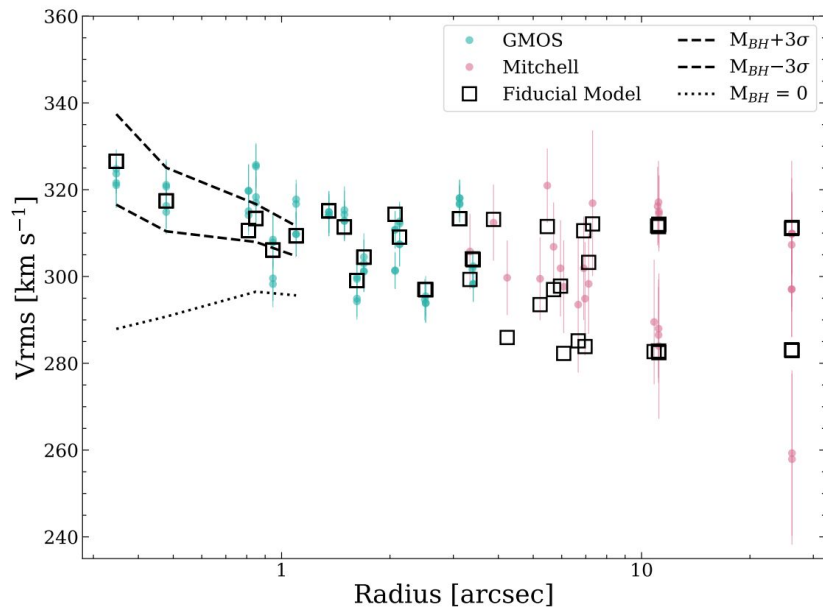
- first simultaneous measurements of DM mass, BH mass, and **intrinsic shapes**
- N2693 has a moderately triaxial intrinsic shape,**
 - Intermediate-to-major axis ratio:
 $p = b/a \sim 0.9$
 - Minor-to-major axis ratio: **$q = c/a \sim 0.7$**
- NGC 2693 $M_{\text{BH}} = (1.7 \pm 0.4) \times 10^9 M_{\text{sun}}$**



TRIAXIAL

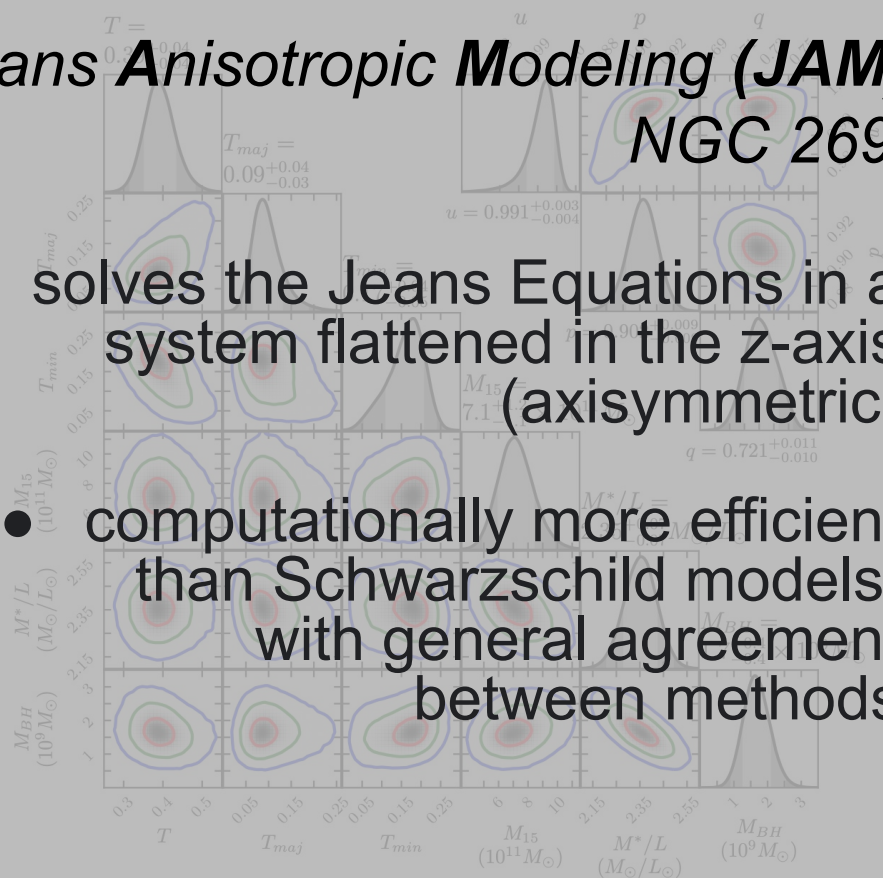
AXISYMMETRIC

NGC 2693 BH: $(1.7 \pm 0.4) \times 10^9 M_{\text{sun}} \rightarrow (2.4 \pm 0.6) \times 10^9 M_{\text{sun}}$



Jeans **Anisotropic Modeling (JAM):** NGC 2693

- solves the Jeans Equations in a system flattened in the z-axis (axisymmetric)
- computationally more efficient than Schwarzschild models, with general agreement between methods



- NGC 2693 $M_{BH} = (1.7 \pm 0.4) \times 10^9 M_{sun}$

JAM

AXISYMMETRIC Schwarzschild

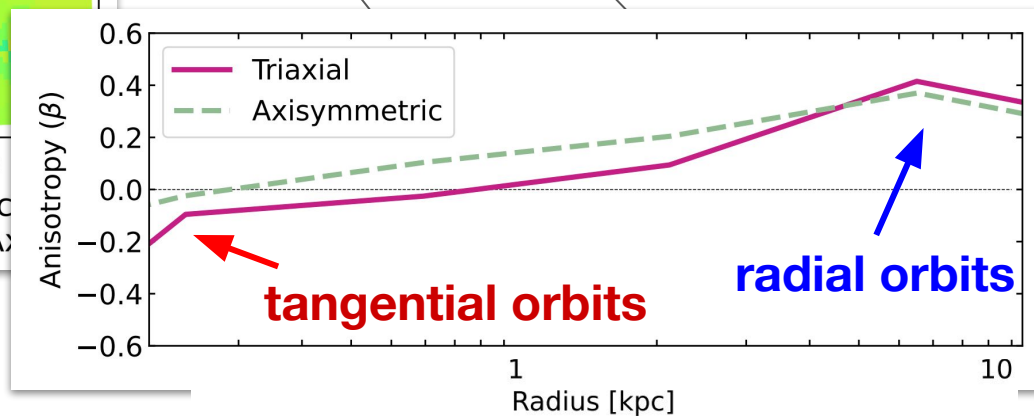
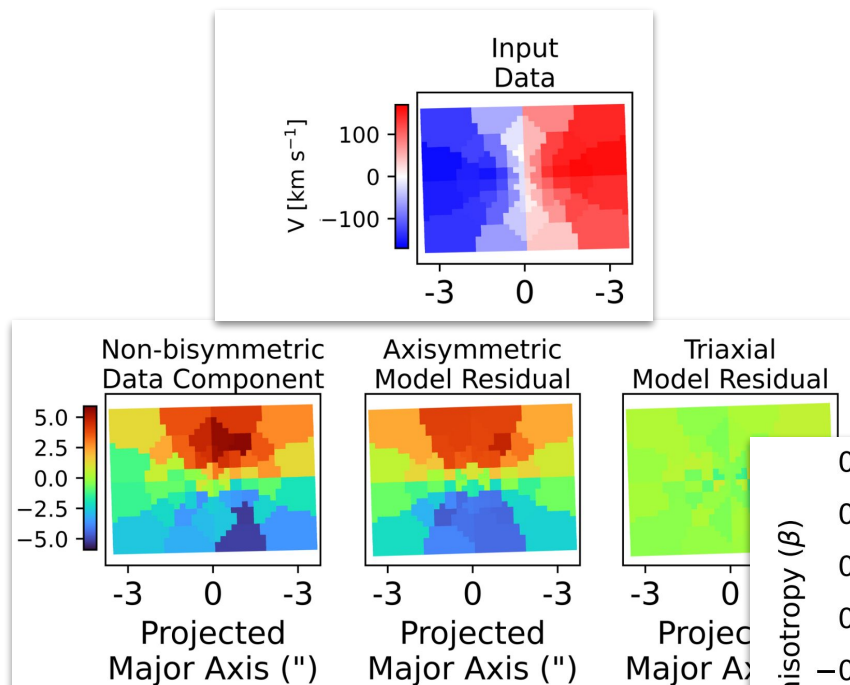
$$(2.9 \pm 0.3) \times 10^9 M_{sun} \rightarrow (2.4 \pm 0.6) \times 10^9 M_{sun}$$

what happened to the axisymmetric models?

analogy:

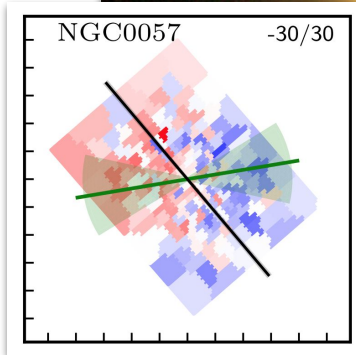
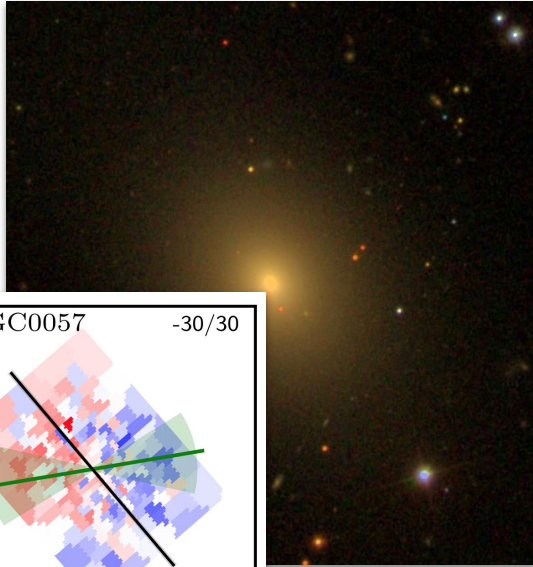
$$f(x) = f_{\text{even}}(x) + f_{\text{odd}}(x)$$

$$V(x) = V_{\text{bisymmetric}}(x) + V_{\text{non-bisymmetric}}(x)$$



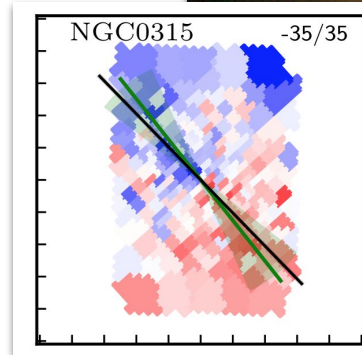
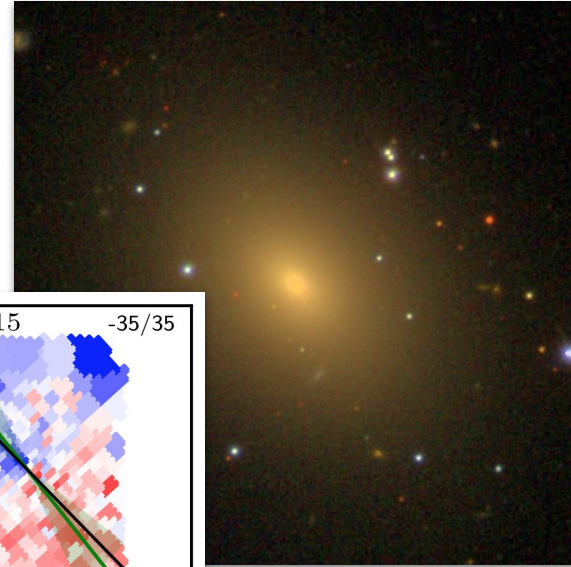
stellar dynamical modeling: ongoing work

NGC 57



- Slow rotation (~ 25 km/s), **kinematically** and **photometrically** misaligned
- Most isolated MASSIVE galaxy

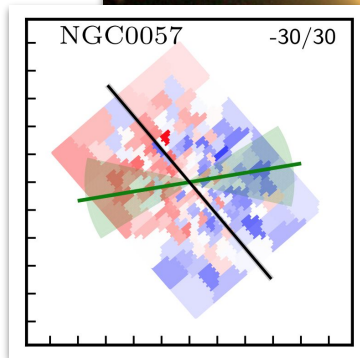
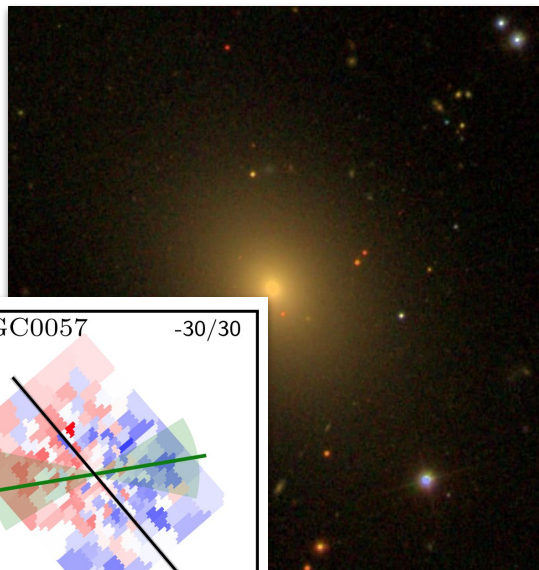
NGC 315



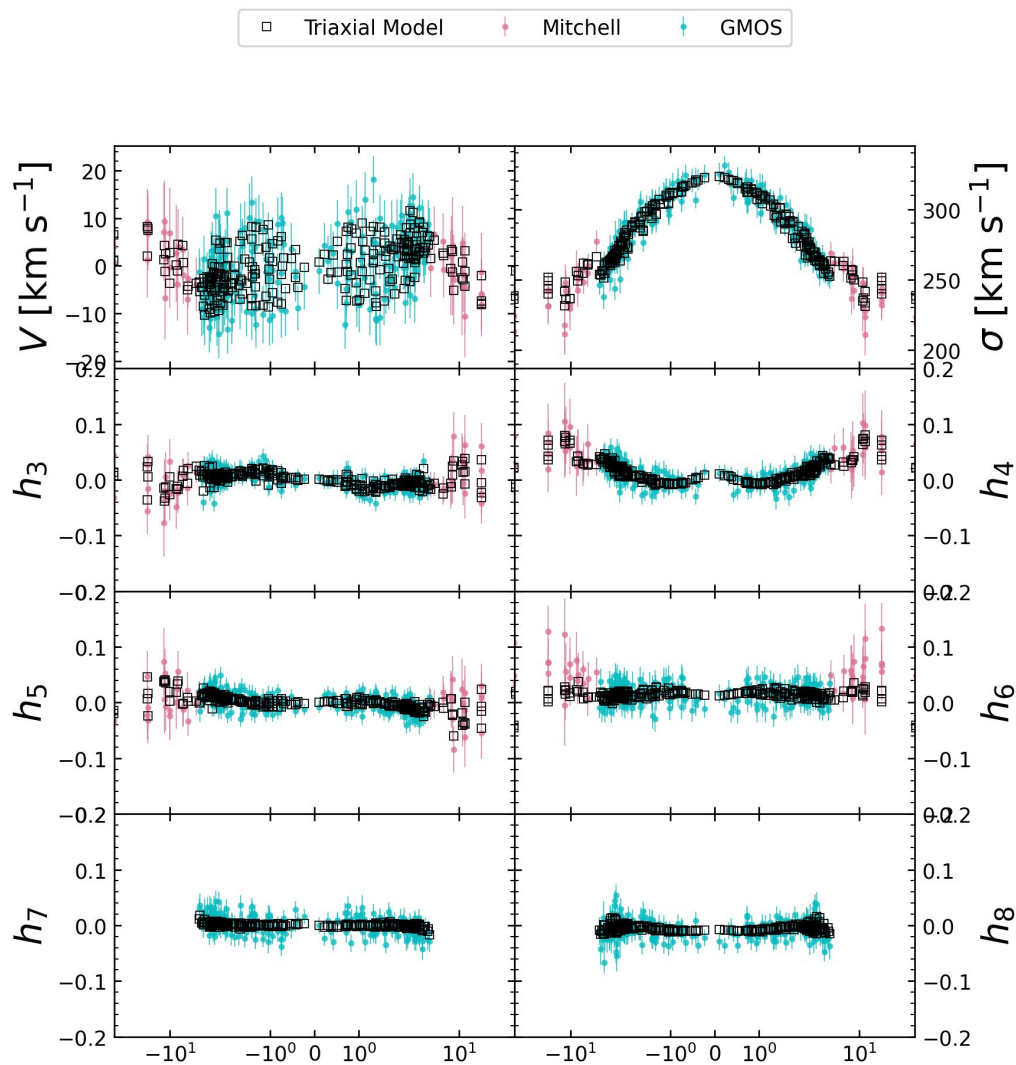
- Slow rotation (~ 30 km/s), **kinematically** and **photometrically** aligned
- Has ALMA CO M_{BH} measurement [Boizelle, Walsh+2021]

stellar dynamical modeling: ongoing work

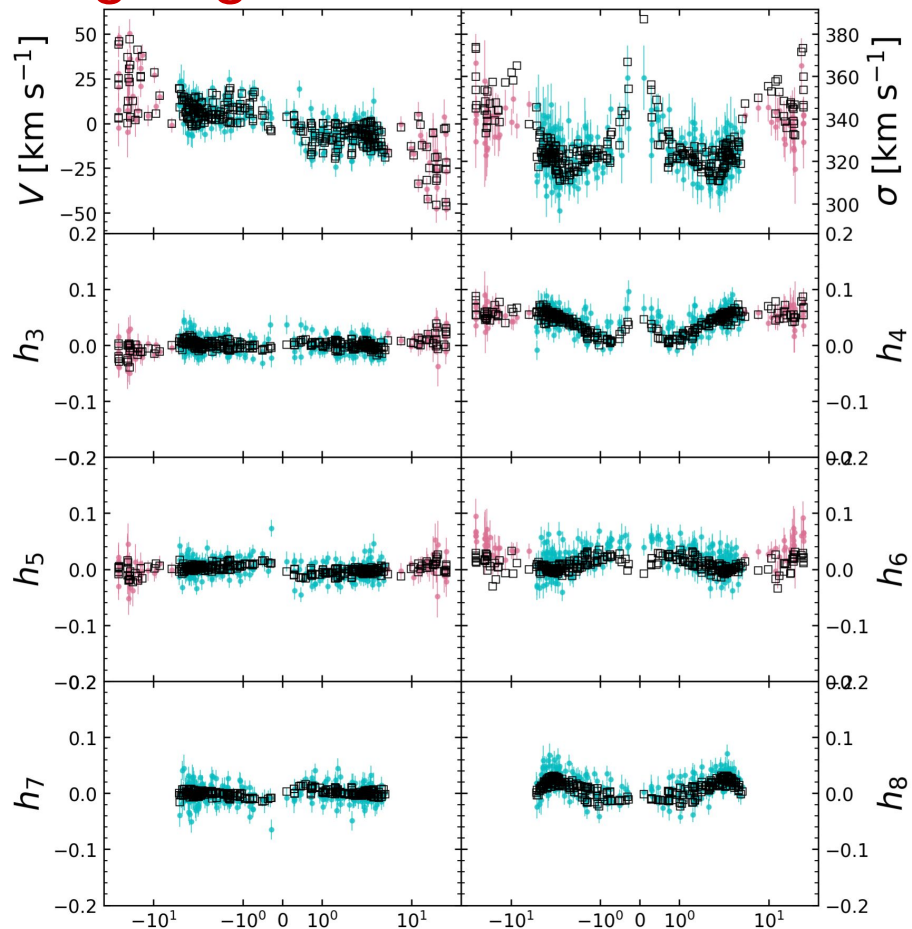
NGC 57



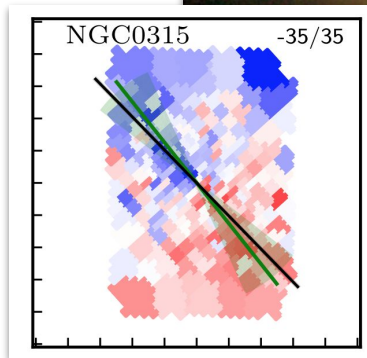
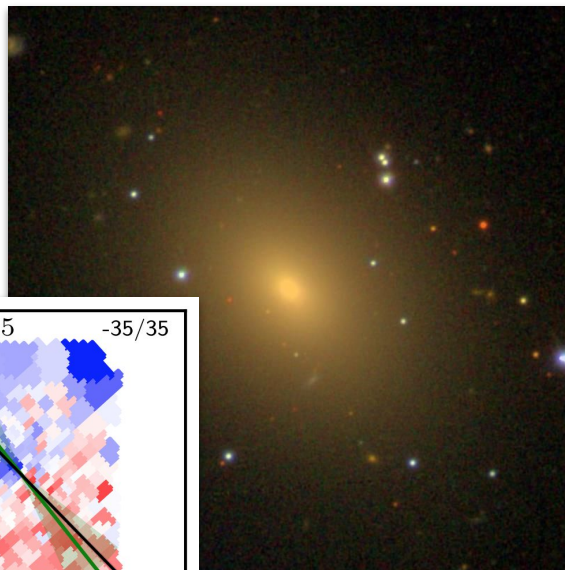
- Slow rotation (~ 25 km/s), **kinematically** and **photometrically** misaligned
- Most isolated MASSIVE galaxy



stellar dynamical modeling: ongoing work



NGC 315



- Slow rotation (~ 30 km/s), **kinematically** and **photometrically** aligned
- Has ALMA CO M_{BH} measurement

A roadmap for the talk:

1. Supermassive black holes in the largest galaxies
2. Triaxial Schwarzschild modeling of massive ellipticals
3. Robustness and Recovery Tests of Dynamical Models



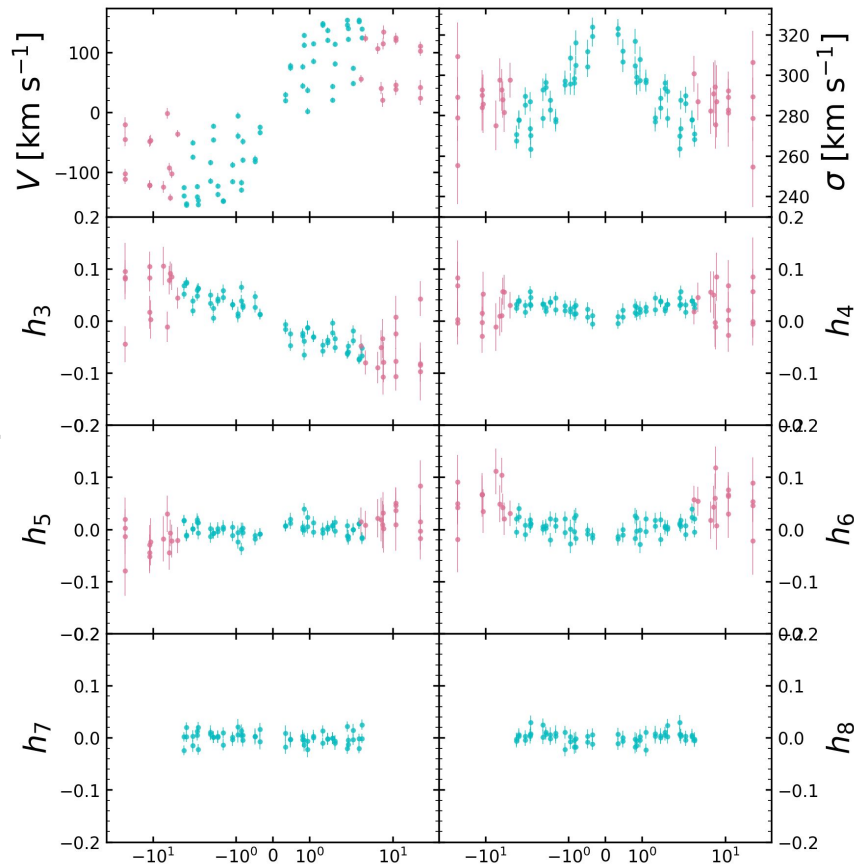
Taking a step back...

how robust are
Schwarzschild
orbit models?

how well can we
recover a series of
known inputs?

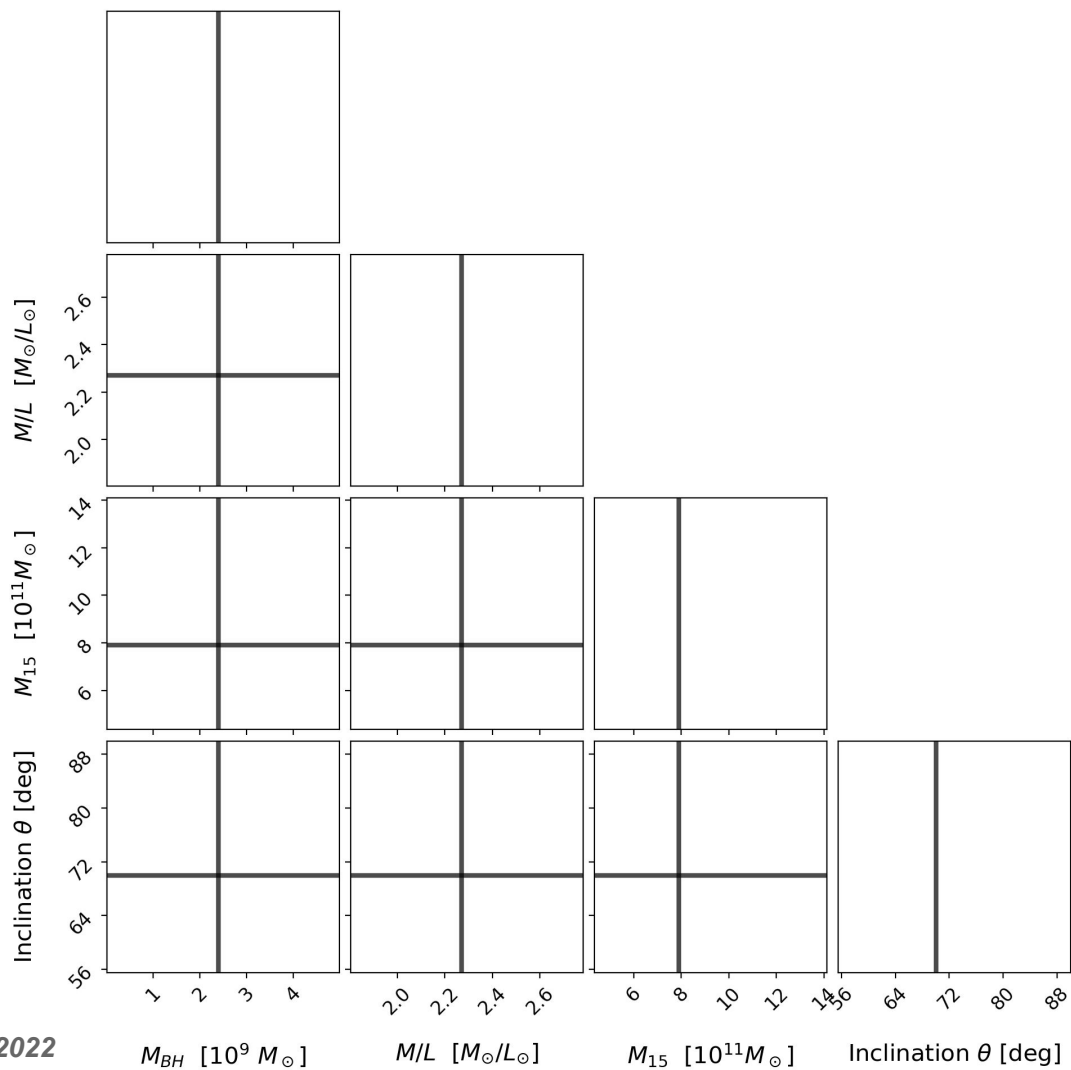
TriOS

input: mock kinematics from a
galaxy with known parameters



Schwarzschild Mock Recovery: Axisymmetric Models

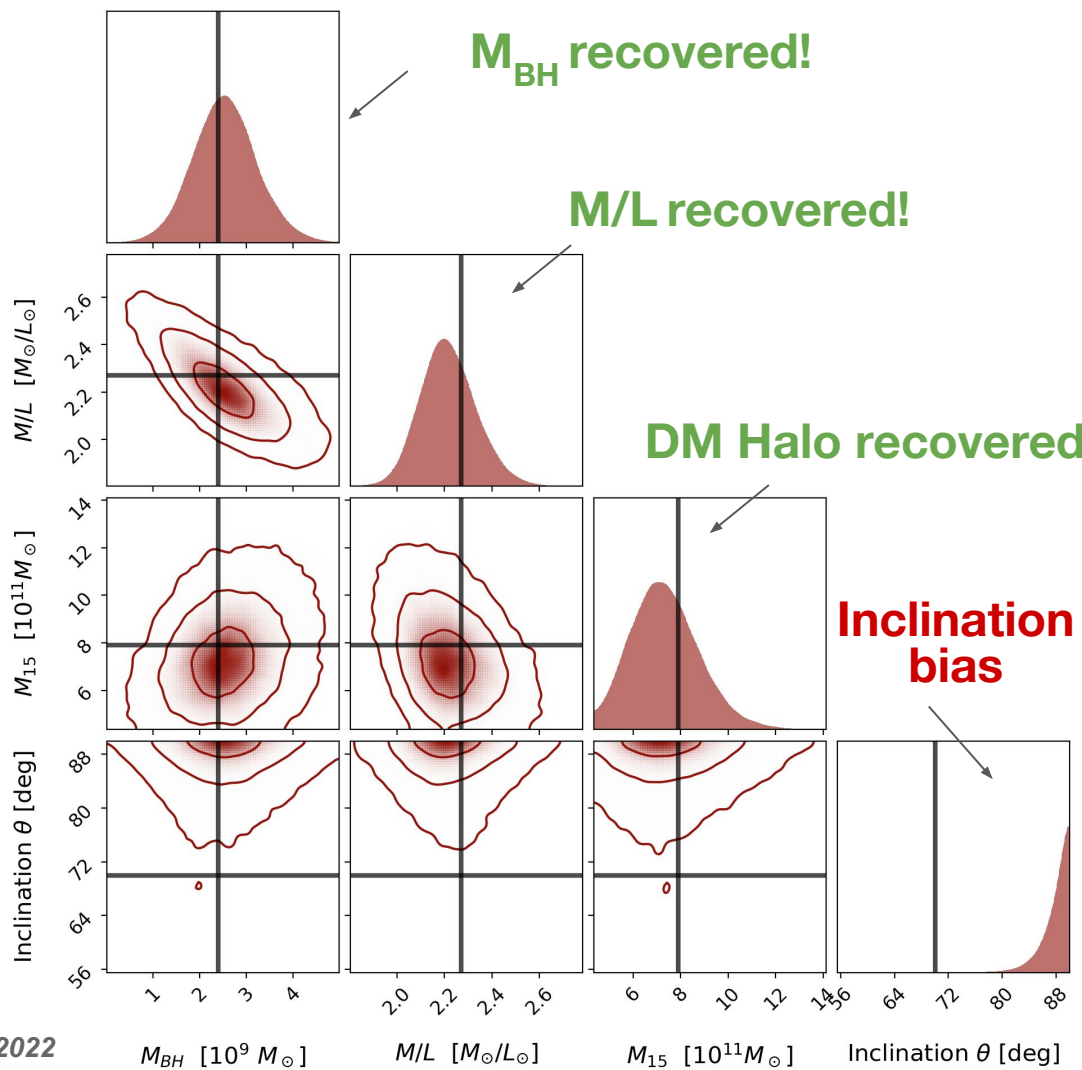
black:
input model kinematics
(from an $i = 70^\circ$ mock
galaxy model)



Schwarzschild Mock Recovery: Axisymmetric Models

black:
input model kinematics
(from an $i = 70^\circ$ mock
galaxy model)

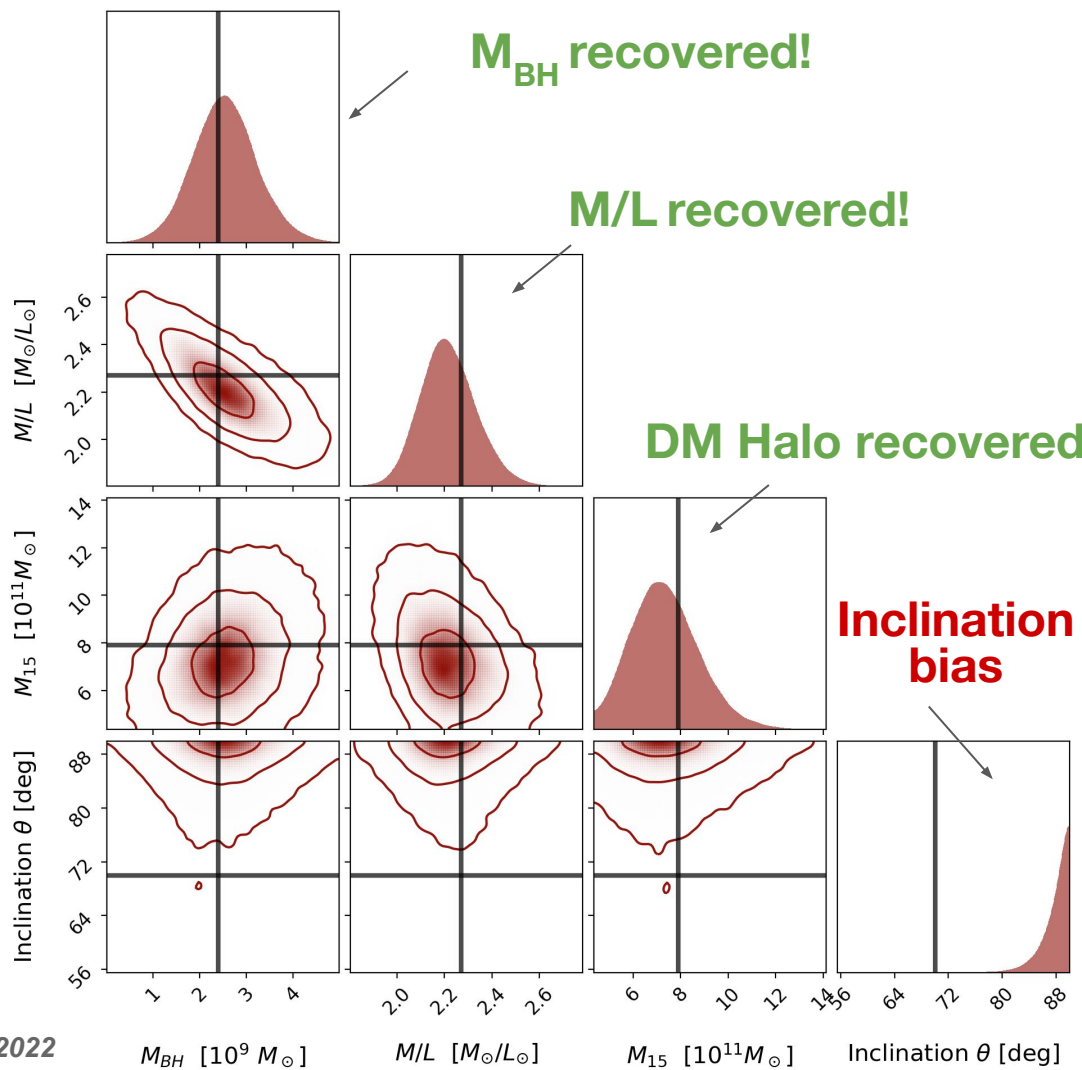
red: recovered posteriors
on parameters from
Schwarzschild models



Schwarzschild Mock Recovery: Axisymmetric Models

best case scenario:
large range of inclinations
are consistent with the data

typical case:
axisymmetric
schwarzschild
models exhibit a
bias toward
edge-on
inclinations

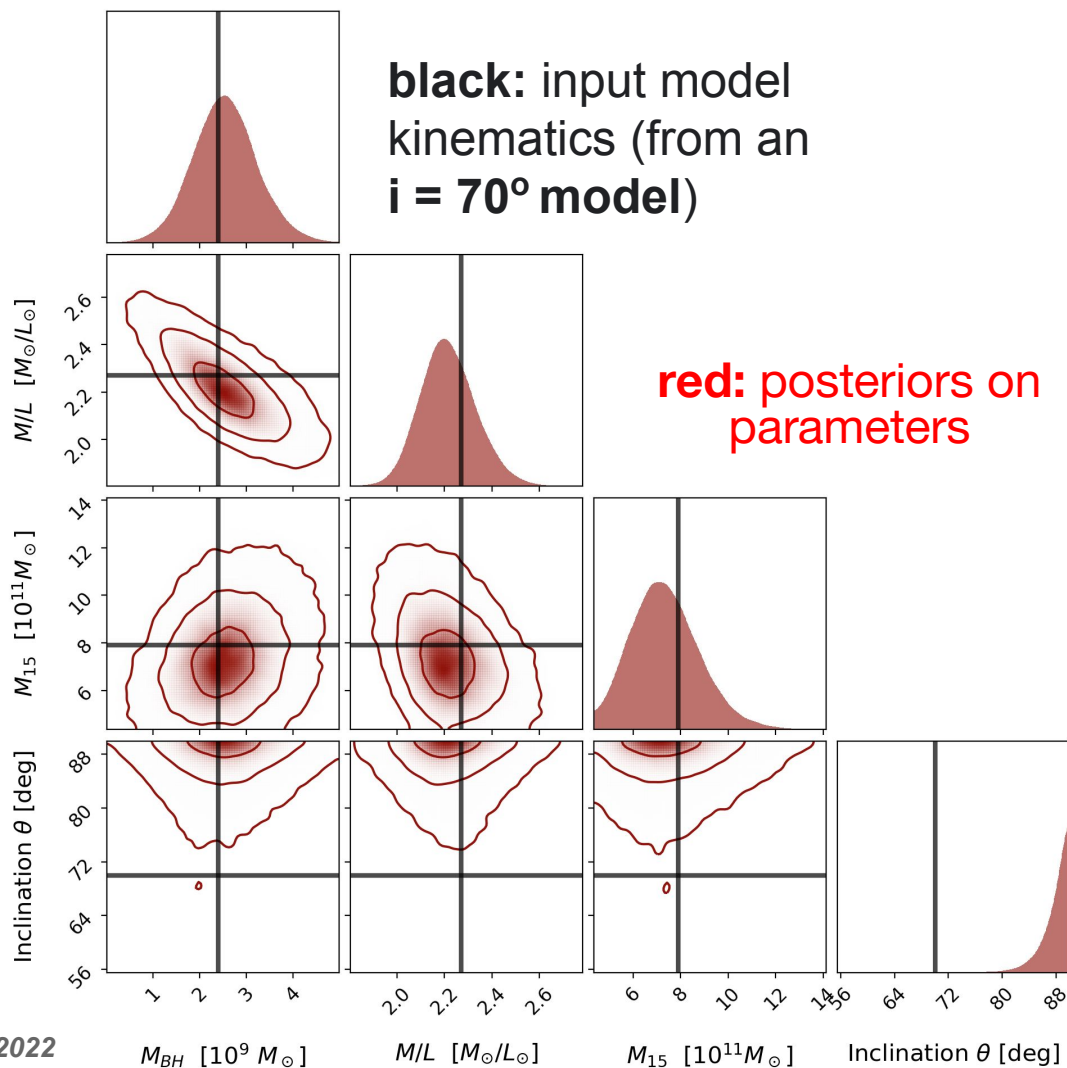


Schwarzschild Model Flexibility: Axisymmetric Models

edge on:
prograde and retrograde
orbits are maximally
different

face on:
prograde and retrograde
orbits are identical

→ edge-on models have a
larger set of unique **basis
functions** for the
superposition of orbits!
→ larger “**flexibility**” in
these models



Schwarzschild Mock Recovery: Axisymmetric Models

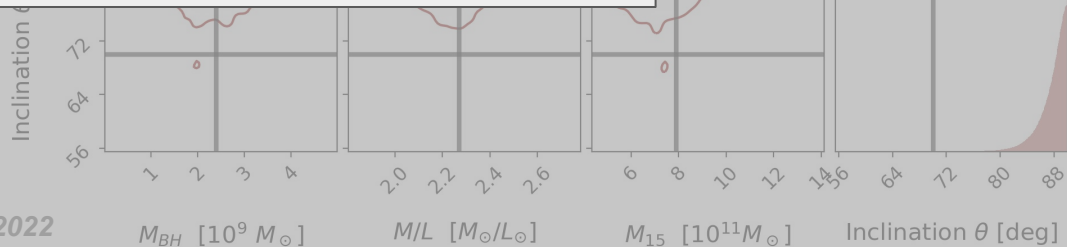
best case
large range of
are consistent

typical
axisym
schwa
models
bias toward
edge-on
inclinations

black: input model
kinematics (from an
 $i = 70^\circ$ model)

- (a) how well can we recover a series of known inputs from our **triaxial** models?
- (b) are there analogous biases in our triaxial modeling scheme?

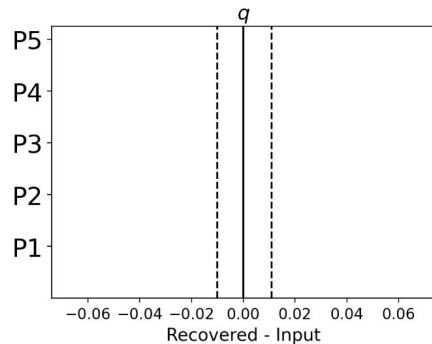
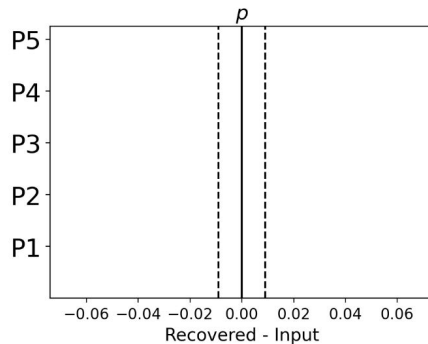
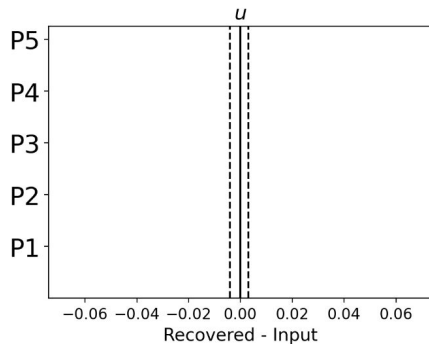
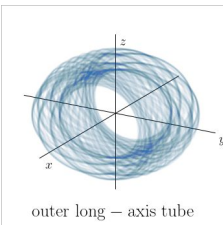
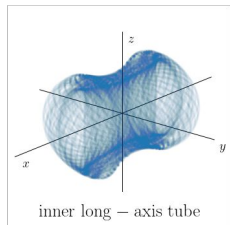
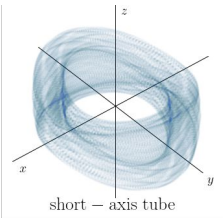
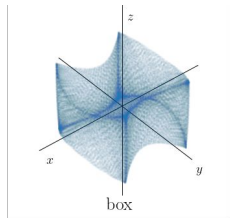
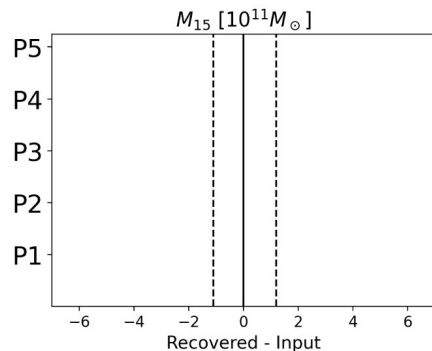
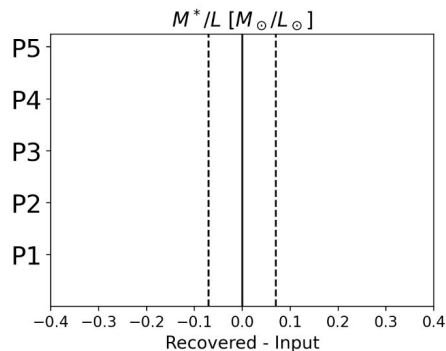
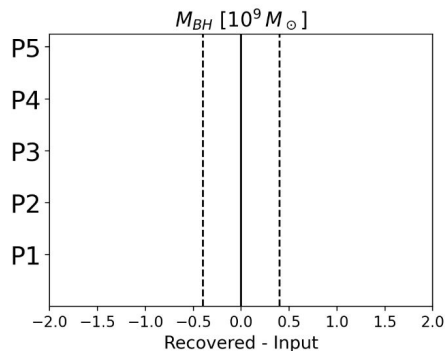
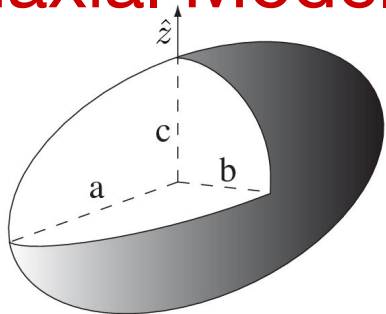
red: posteriors on parameters



Schwarzschild Model

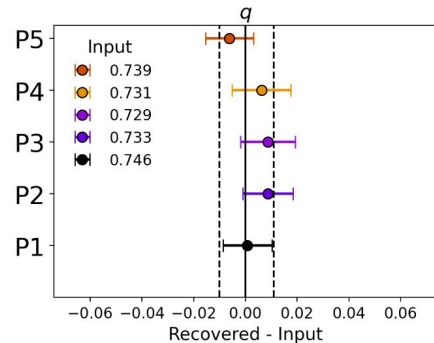
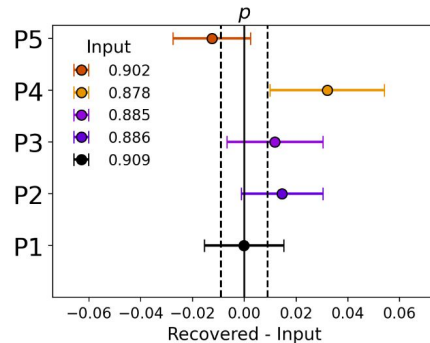
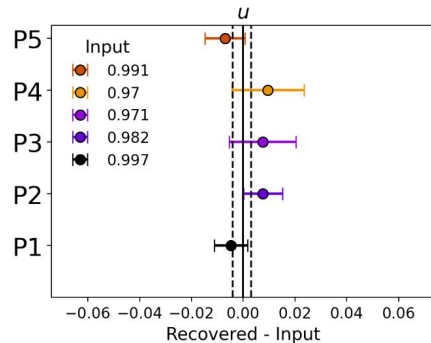
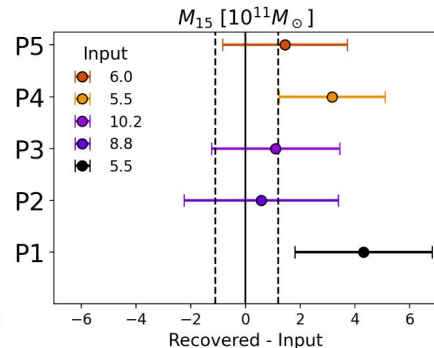
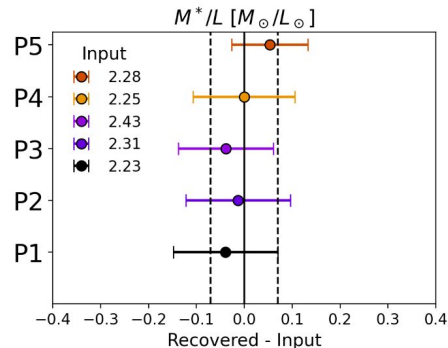
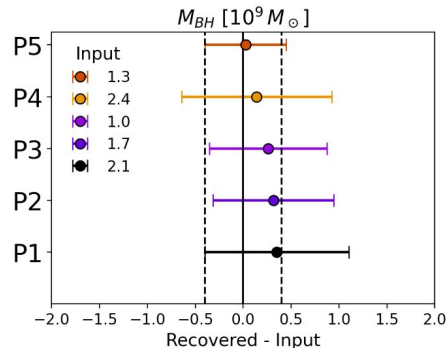
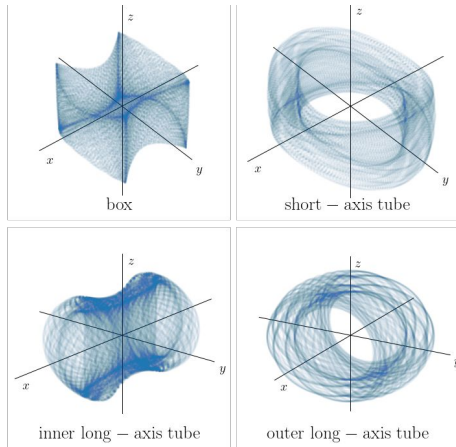
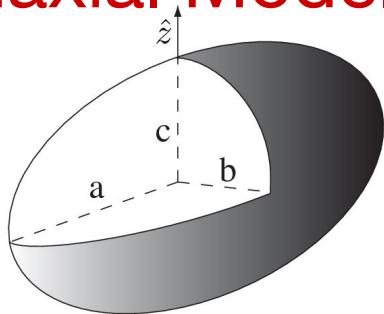
Flexibility:

Triaxial Models



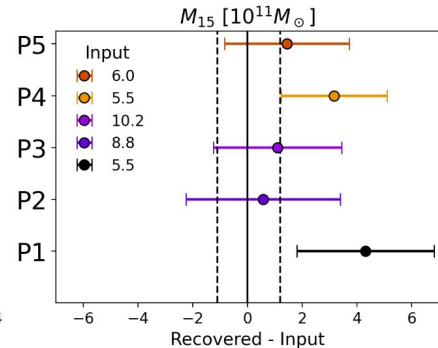
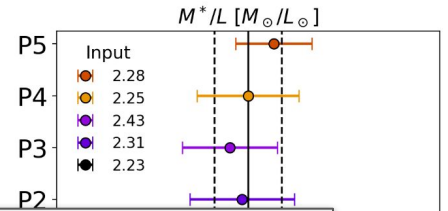
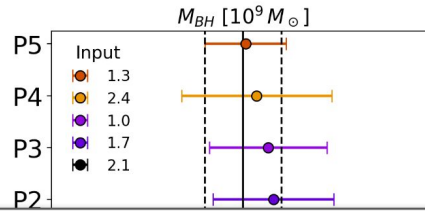
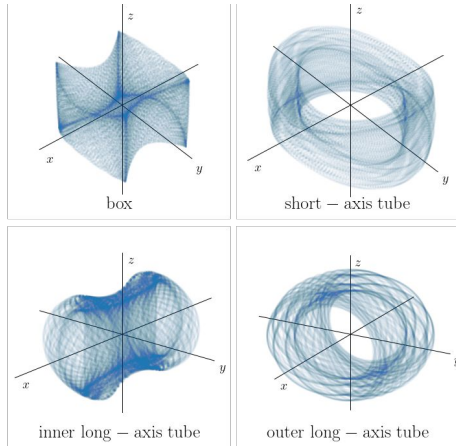
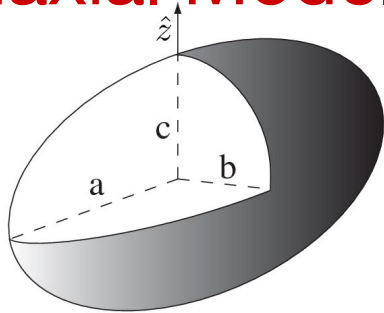
Schwarzschild Model

Flexibility: Triaxial Models

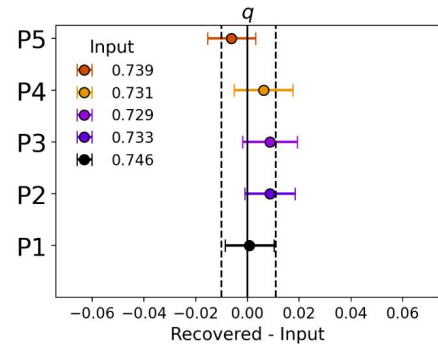
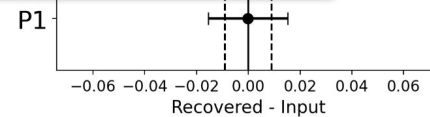
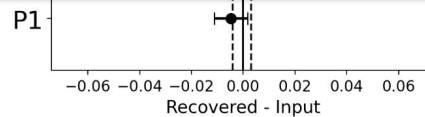


Schwarzschild Model

Flexibility: Triaxial Models

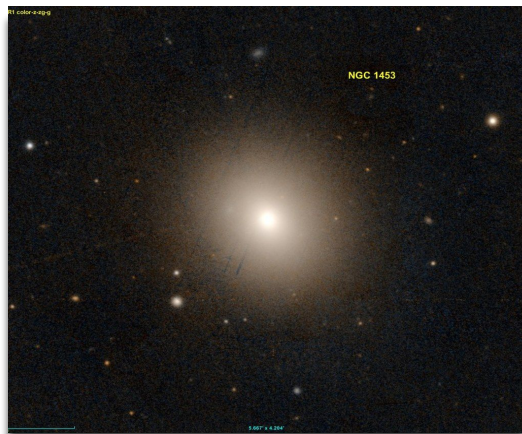


Triaxial models
do not seem to exhibit
the same biases as
axisymmetric models.



Summary: Recent Dynamical Modeling Efforts

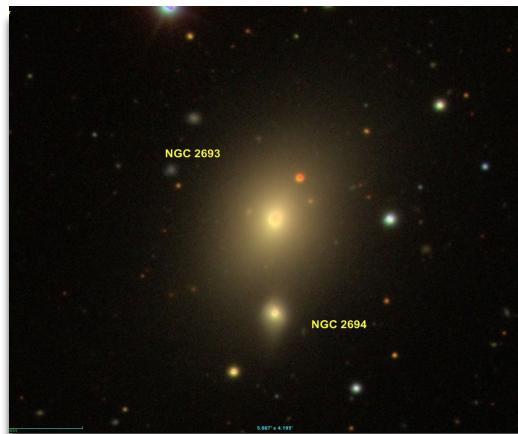
NGC 1453



Quenneville+2021

- $M_{\text{BH}} = (2.9 \pm 0.4) \times 10^9 M_{\text{sun}}$
- $p = b/a = (0.933 \pm 0.015)$
- $q = c/a = (0.779 \pm 0.012)$

NGC 2693



Pilawa+2022

- $M_{\text{BH}} = (1.7 \pm 0.4) \times 10^9 M_{\text{sun}}$
- $p = b/a = (0.902 \pm 0.009)$
- $q = c/a = (0.721 \pm 0.010)$

M87



Liepold+2023

- $(5.37^{+0.37}_{-0.25} \pm 0.22) \times 10^9 M_{\odot}$
- $p = b/a = (0.845 \pm 0.004)$
- $q = c/a = (0.772 \pm 0.004)$

Thank you!

Summary:

discovery and dynamical
modeling of new
supermassive black holes:
NGC 2693: $(1.7 \pm 0.4) \times 10^9 M_{\text{Sun}}$
NGC 1453: $(1.7 \pm 0.4) \times 10^9 M_{\text{Sun}}$

first simultaneous measurements
of **BH + DM halo + galaxy
shape;**
axisymmetric + triaxial + JAM

schwarzschild models are
extremely powerful, and we
can robustly recover known
inputs

computationally cheap ways to
estimate **model flexibility** and its
effects (or lack thereof) on
recovered parameters

we are ready for **more
dynamical modeling** more
complicated kinematic
structure

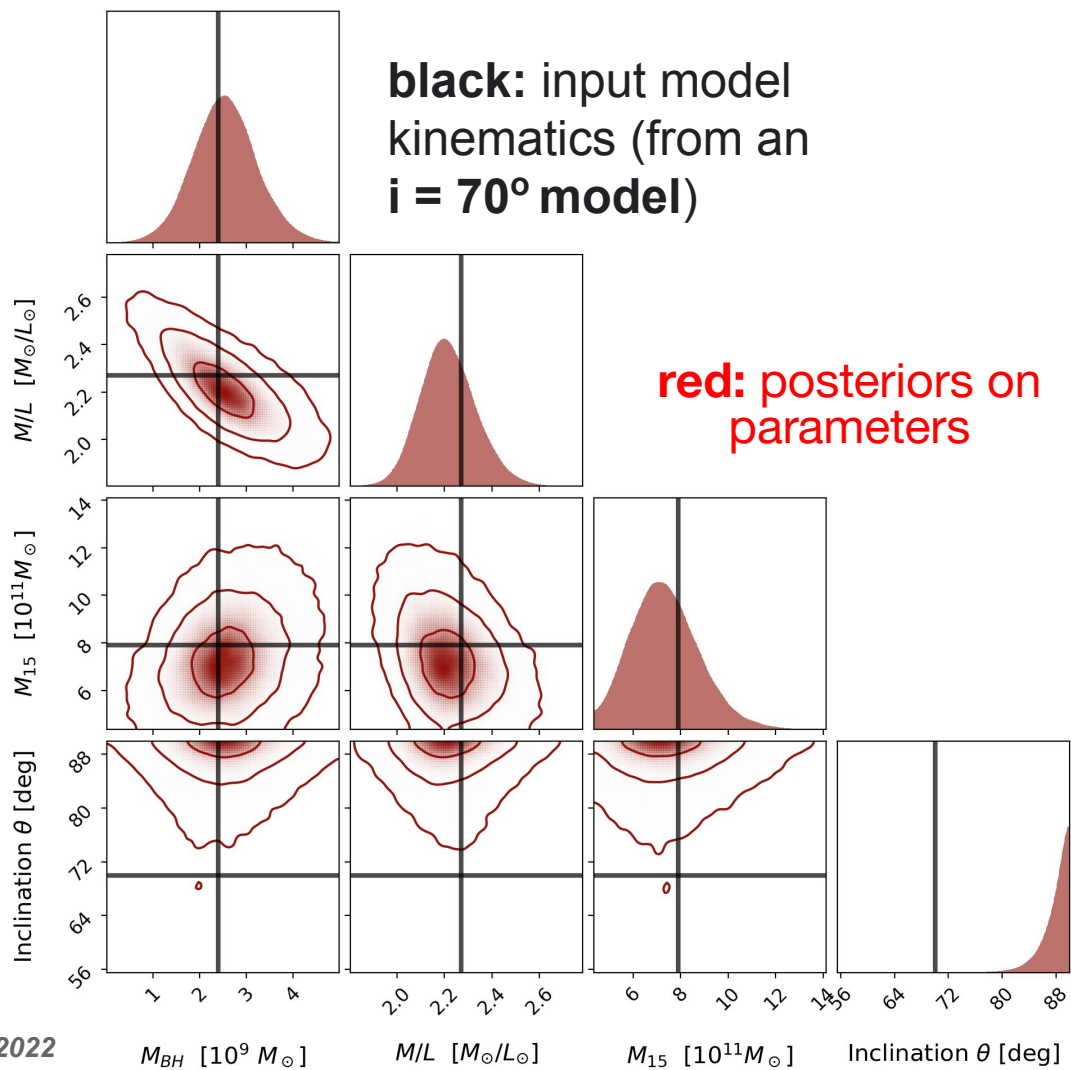
~20 MASSIVE galaxies are
ready for modeling, none of
which are simple fast
rotators

Thank you!

Extra Slides

Schwarzschild Model Flexibility: Axisymmetric Models

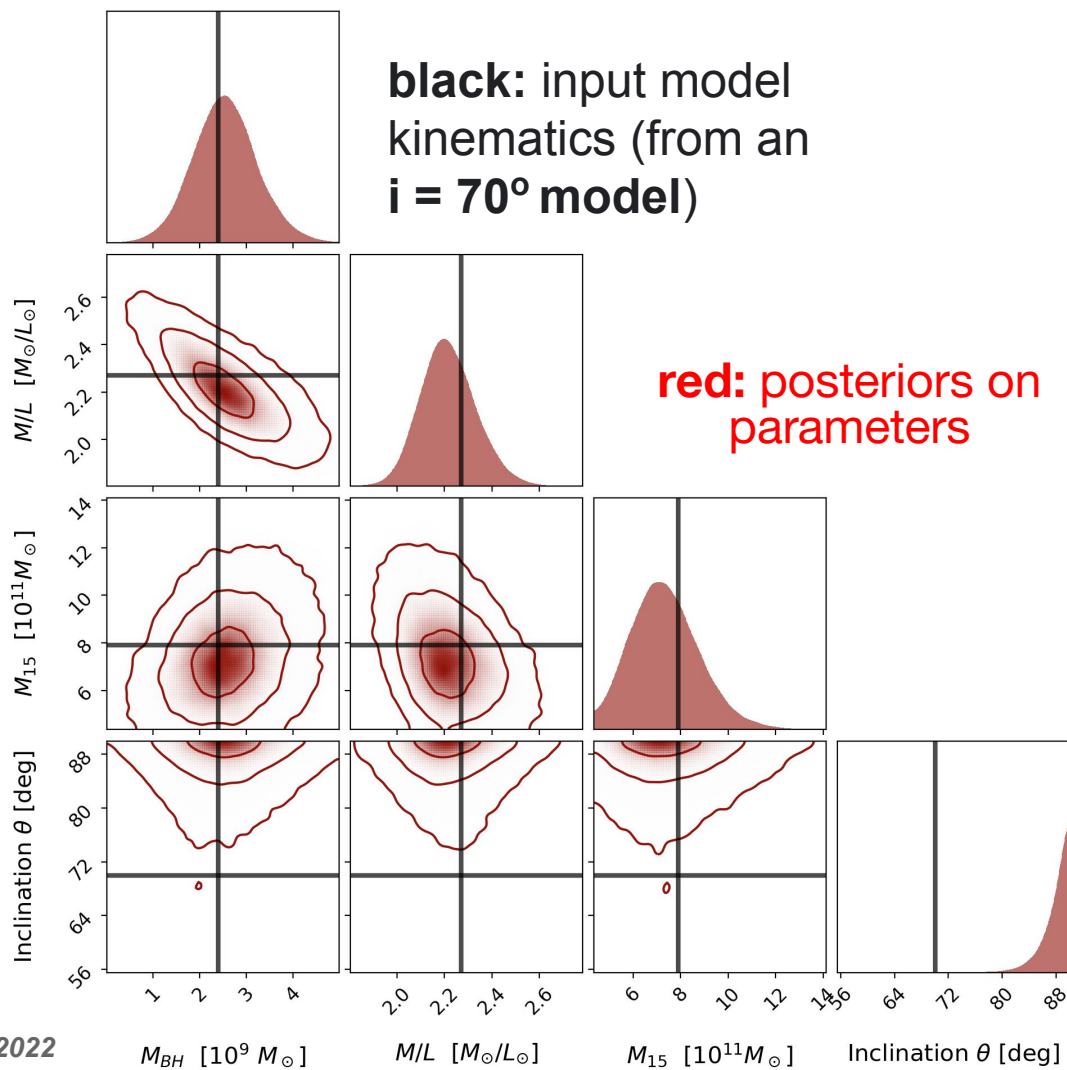
can we fix this?



Schwarzschild Model Flexibility: Axisymmetric Models

can we fix this?

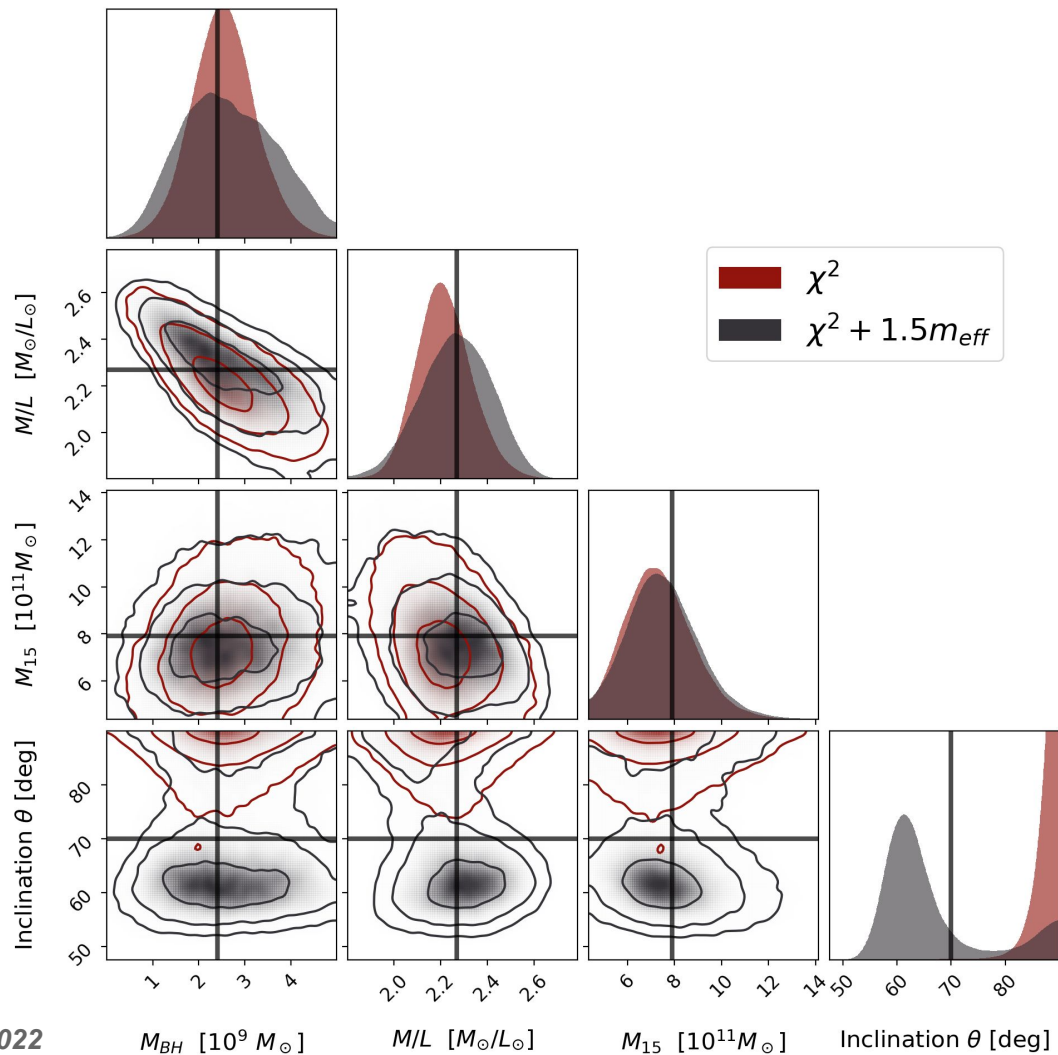
- one approach (Ye 1998):
generalized d.o.f. m_{eff}



Schwarzschild Model Flexibility: Axisymmetric Models

can we fix this?

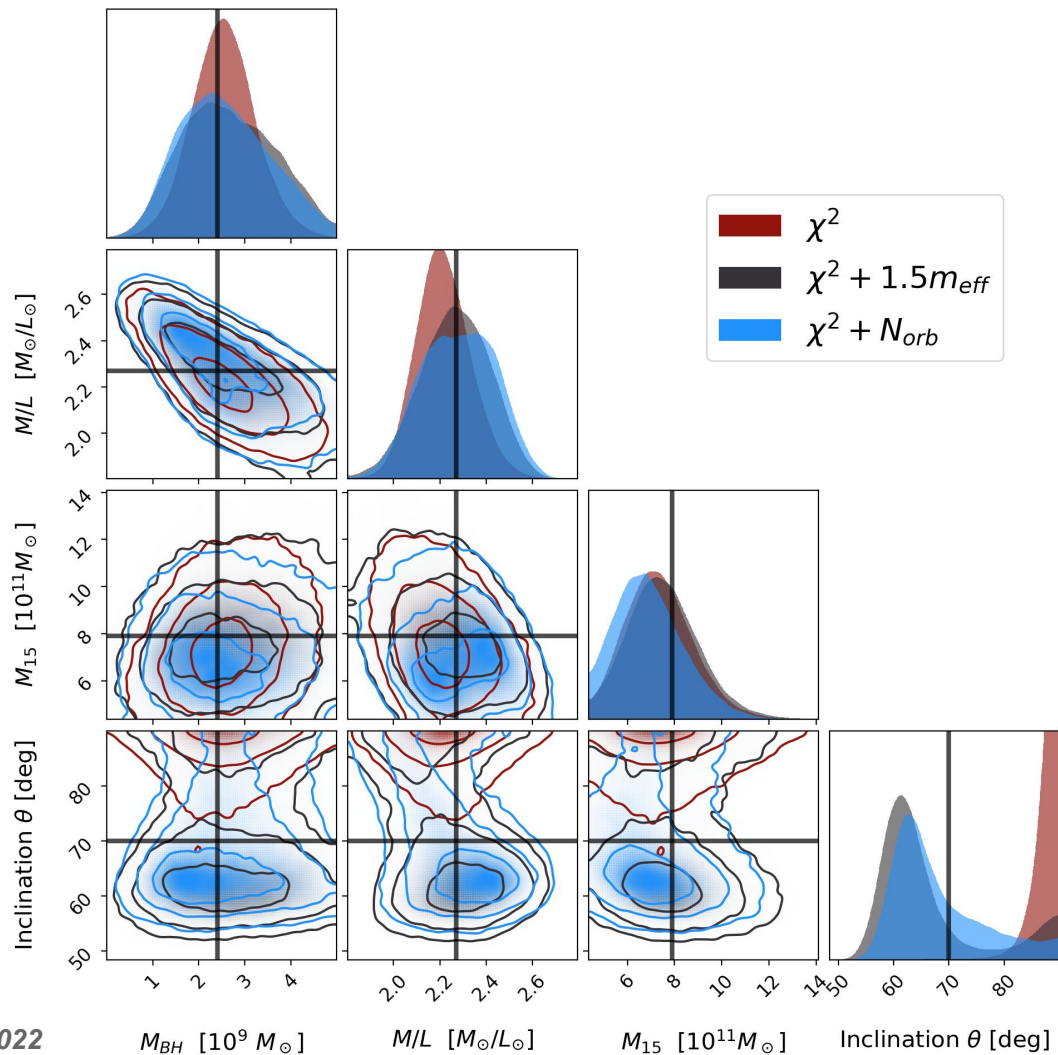
- one approach (Ye 1998):
generalized d.o.f. m_{eff}



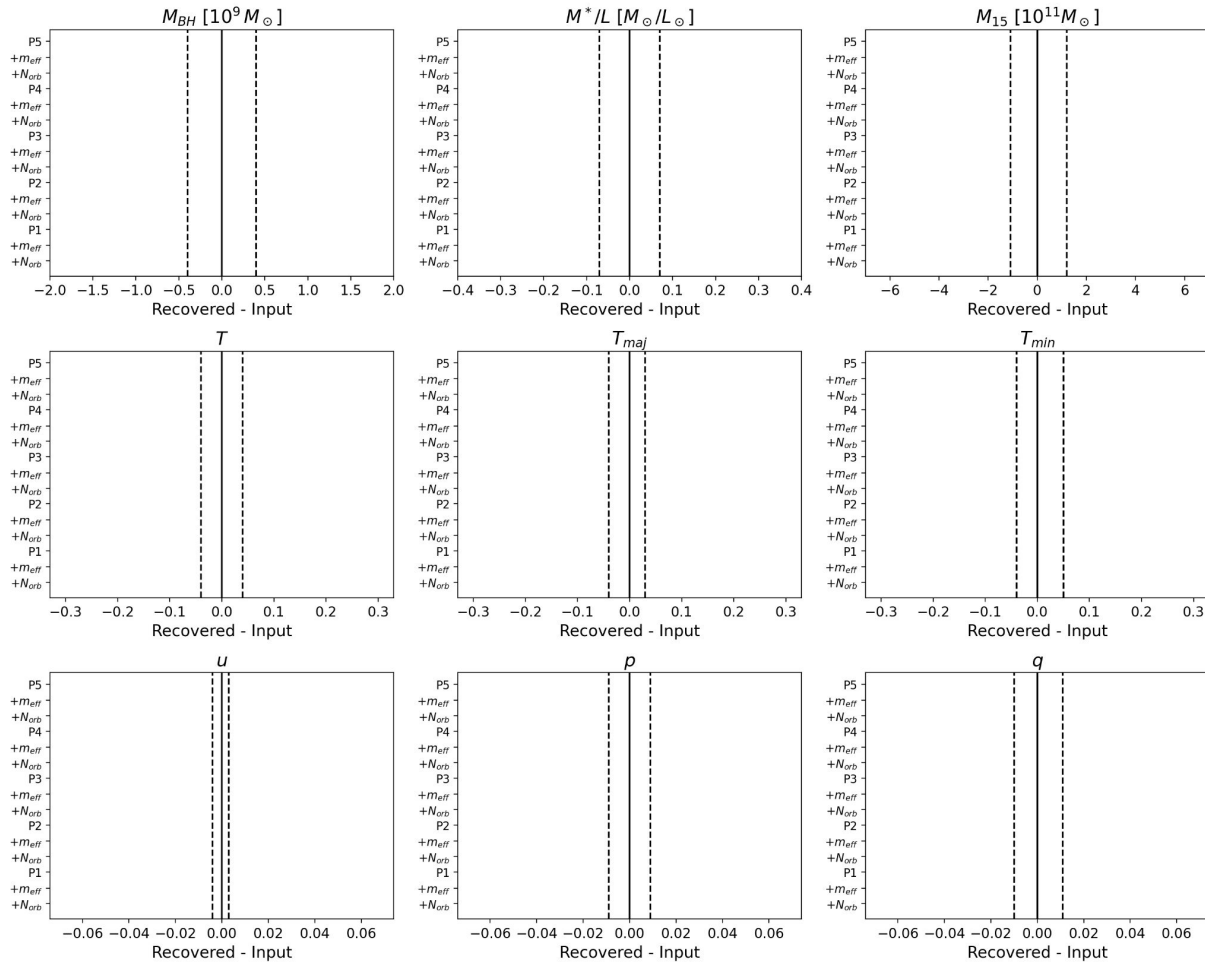
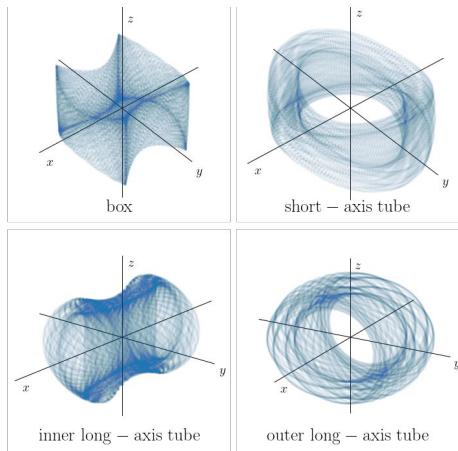
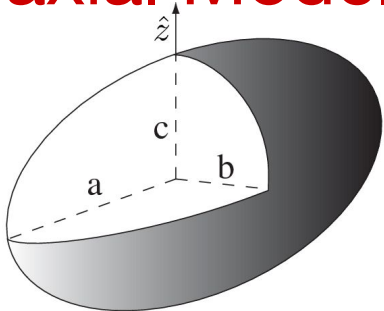
Schwarzschild Model Flexibility: Axisymmetric Models

can we fix this?

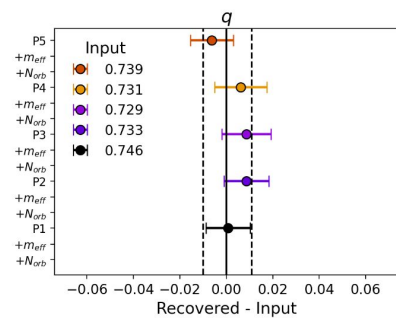
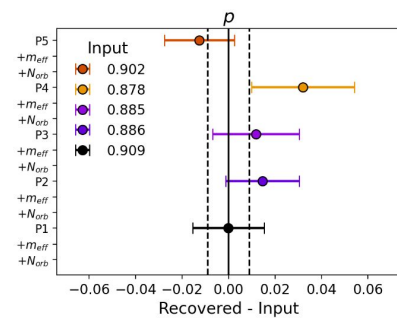
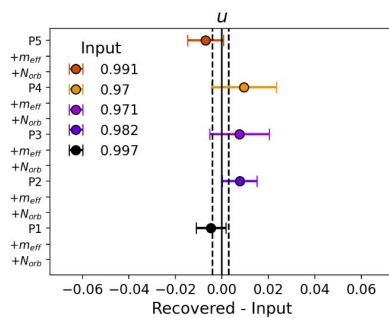
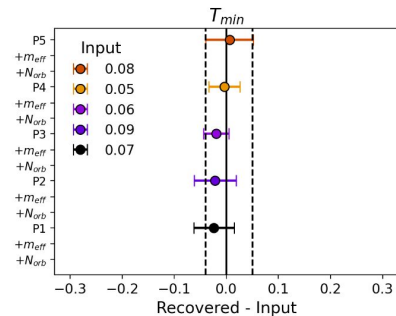
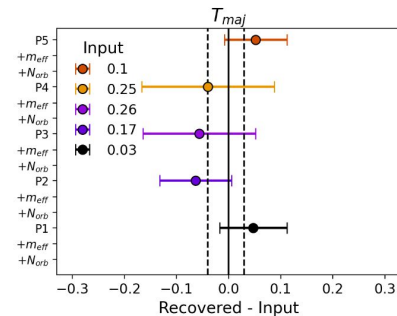
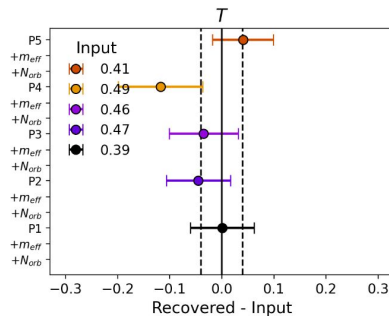
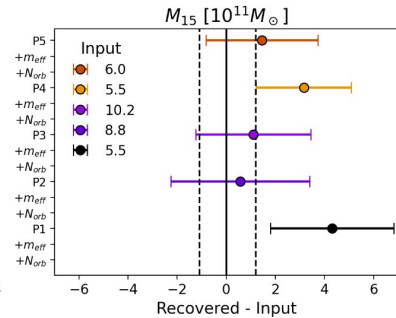
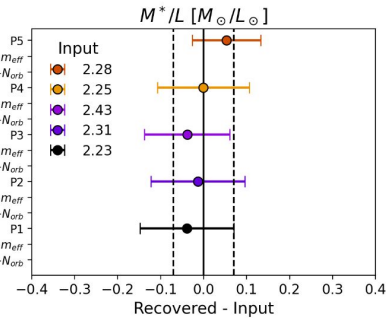
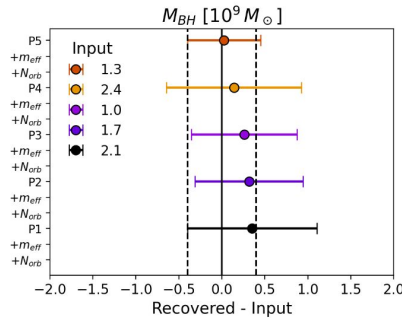
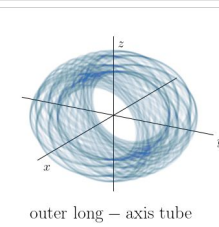
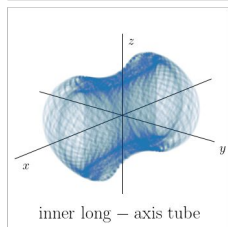
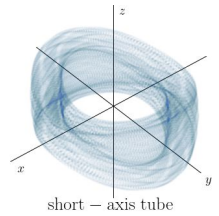
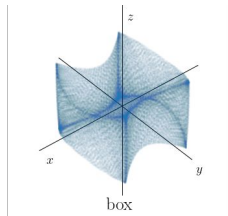
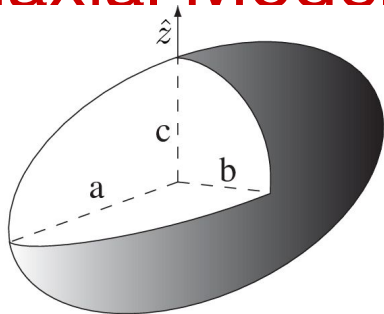
- one approach (Ye 1998):
generalized d.o.f. m_{eff}
- alternative, **faster** approach:
 - penalize the model's χ^2 by “activated” orbits N_{orb} (orbits with non-zero weight in superposition)



Schwarzschild Model Flexibility: Triaxial Models



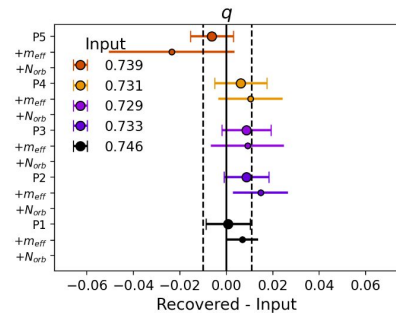
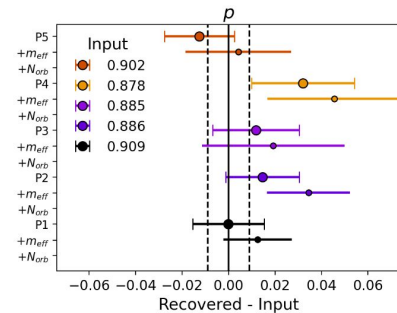
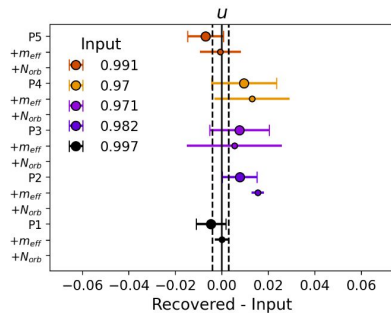
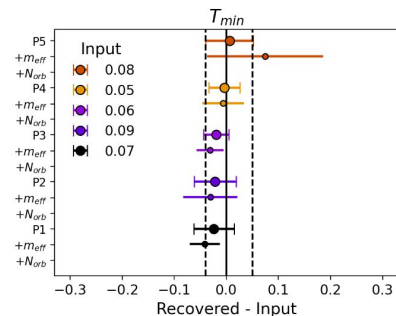
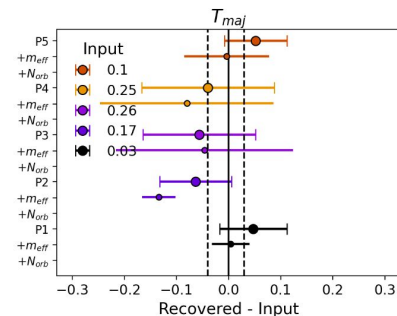
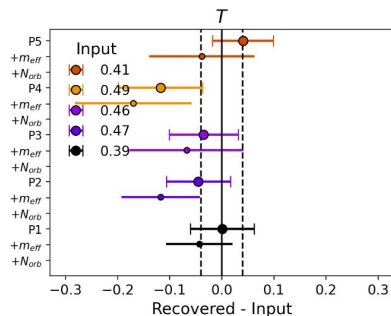
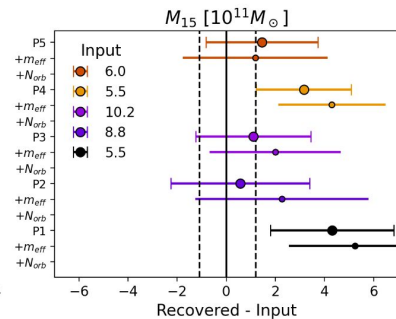
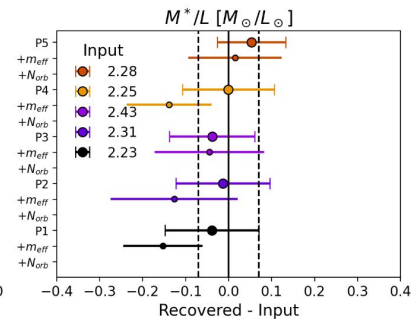
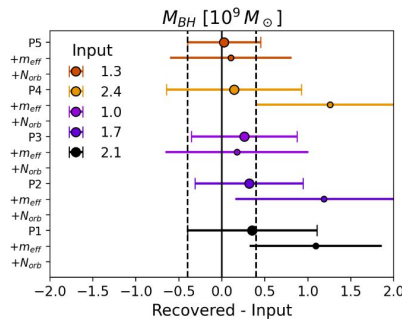
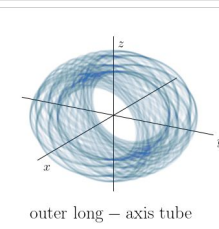
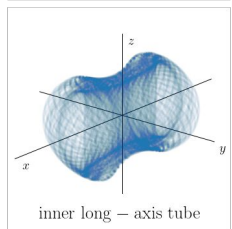
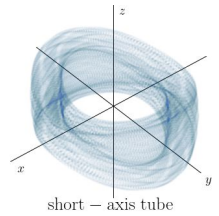
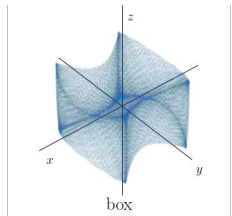
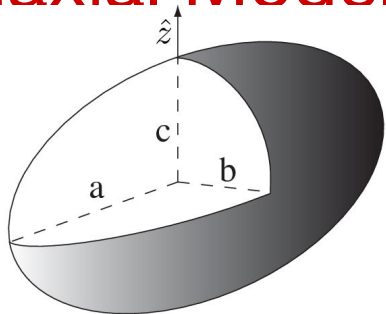
Schwarzschild Model Flexibility: Triaxial Models



Schwarzschild Model

Flexibility:

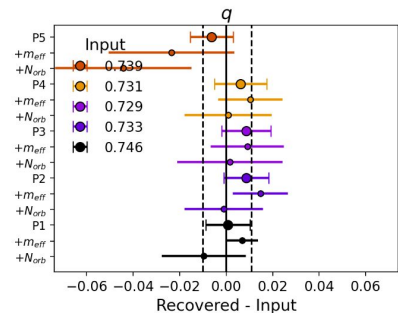
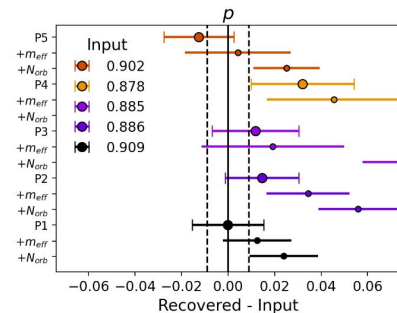
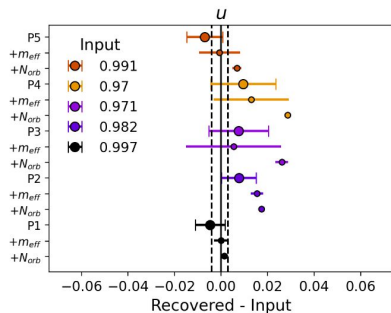
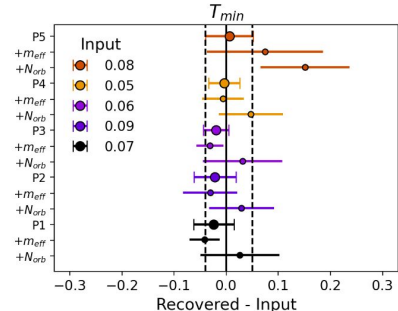
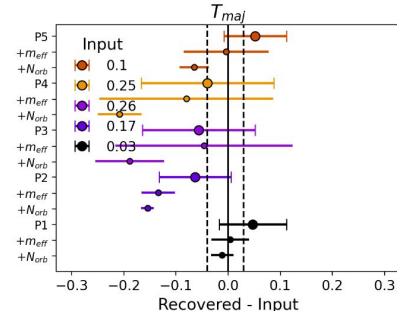
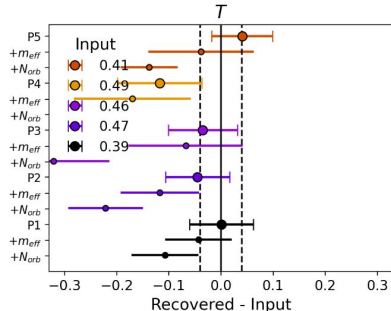
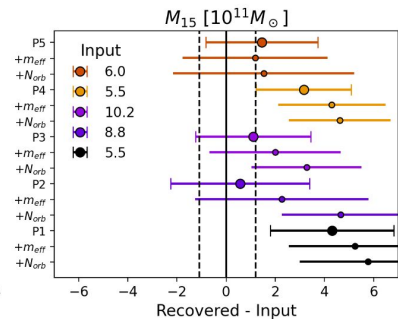
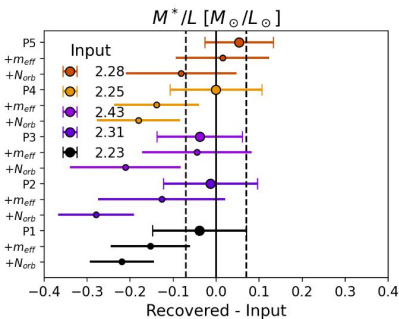
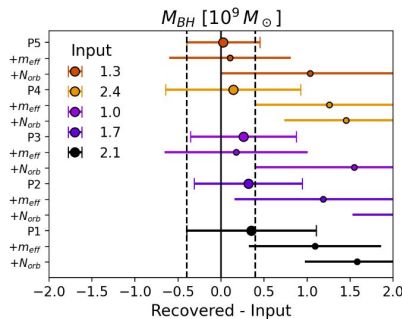
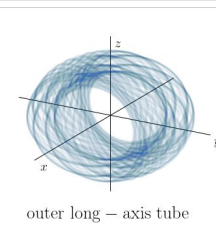
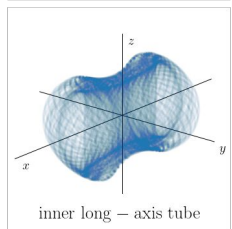
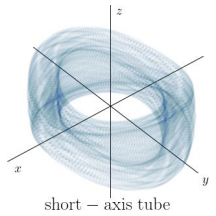
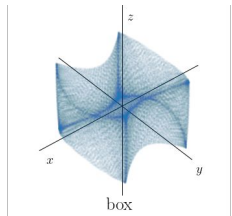
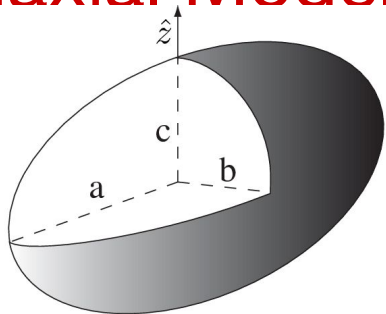
Triaxial Models



Schwarzschild Model

Flexibility:

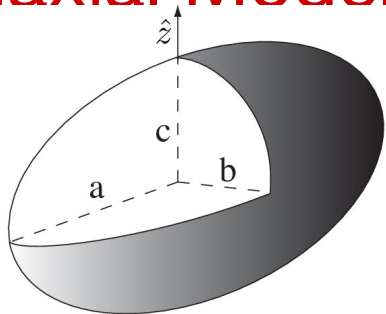
Triaxial Models



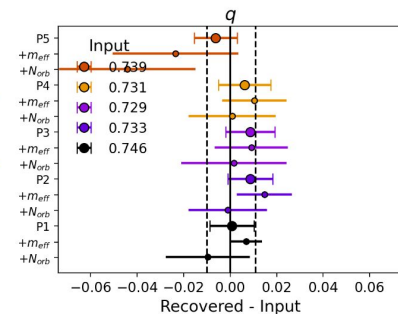
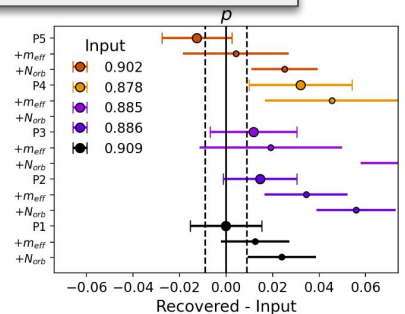
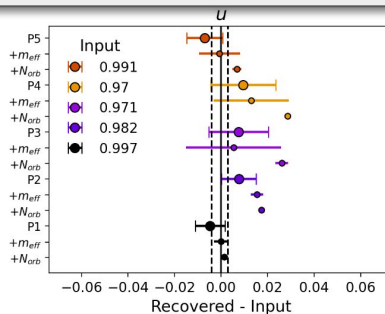
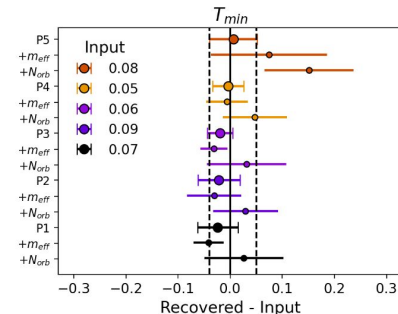
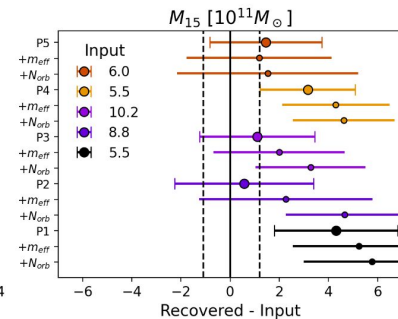
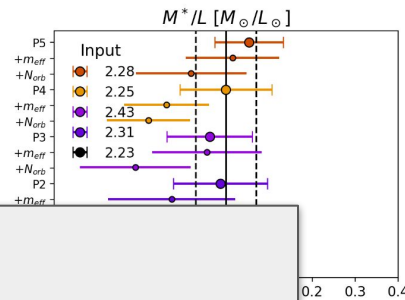
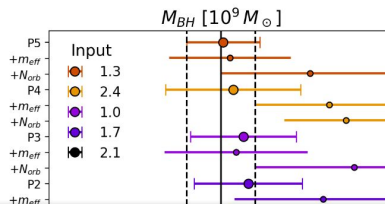
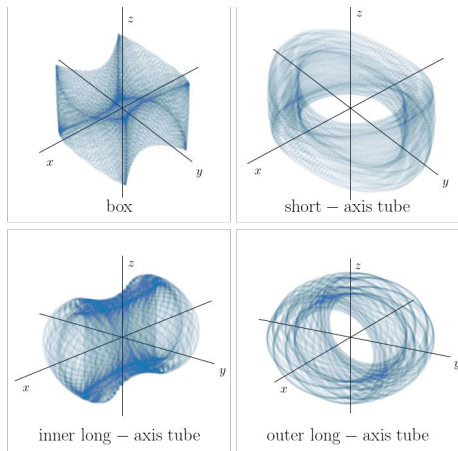
Schwarzschild Model

Flexibility:

Triaxial Models

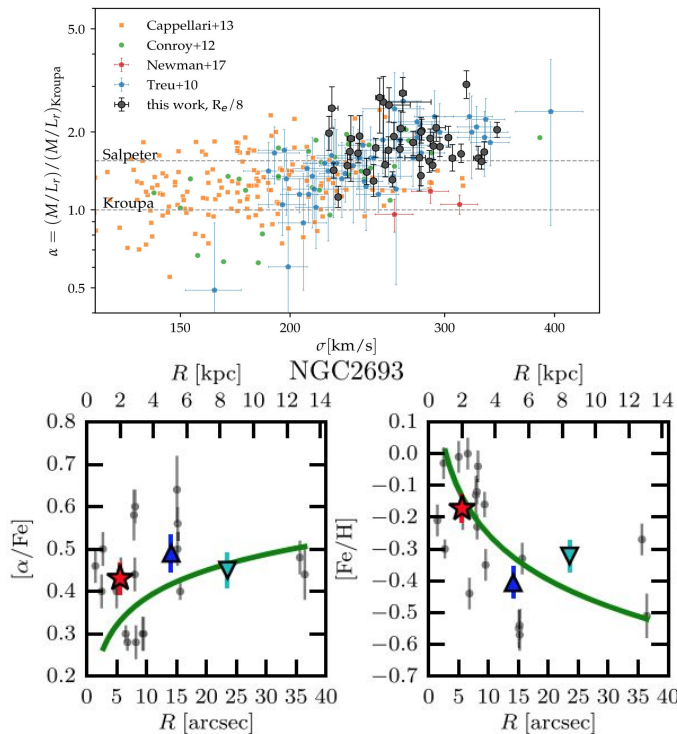


Triaxial models *do not exhibit* the same biases as axisymmetric models.



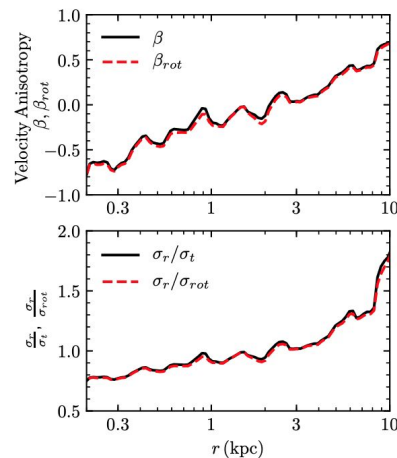
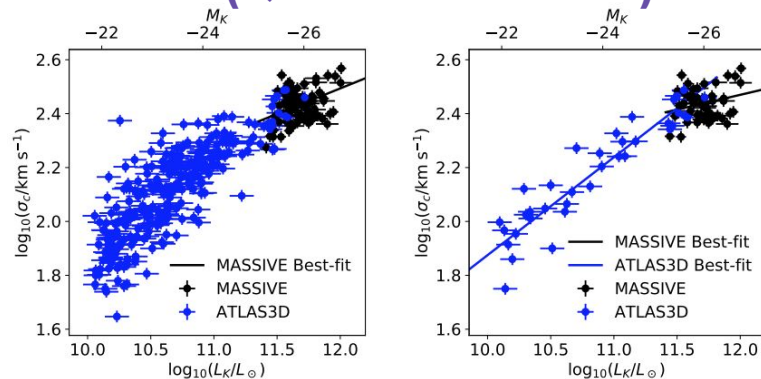
The MASSIVE Survey: Recent Studies

variations in the IMF in massive ellipticals (Gu+2022)



chemical abundance gradients/formation history (Greene+2019)

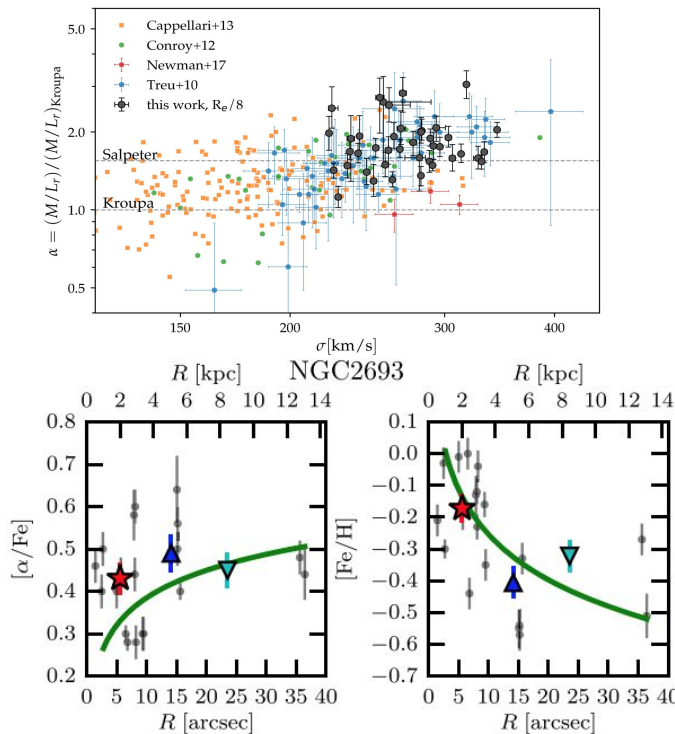
local scaling relations (Quenneville+2022)



dynamical modeling to constrain SMBH masses, shapes (Liepold+2020)

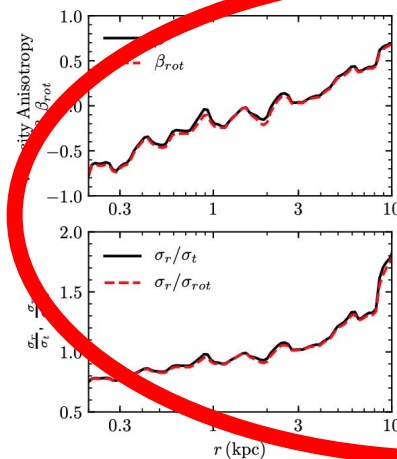
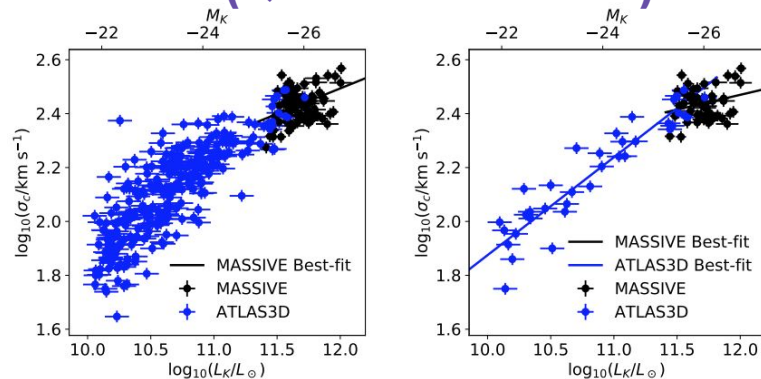
The MASSIVE Survey: Recent Studies

variations in the IMF in massive ellipticals (Gu+2022)



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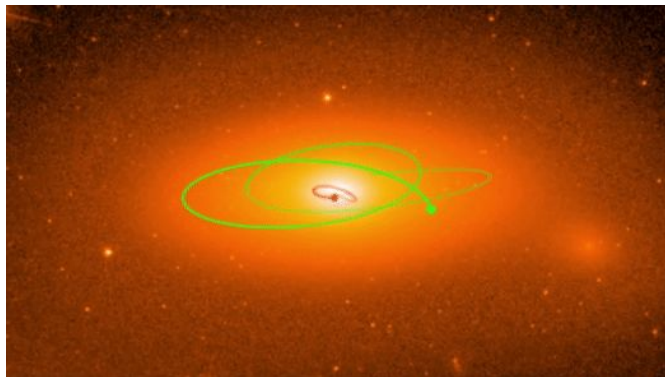


dynamical modeling to constrain SMBH masses, shapes (Liepold+2020)

stellar dynamics pt. 2

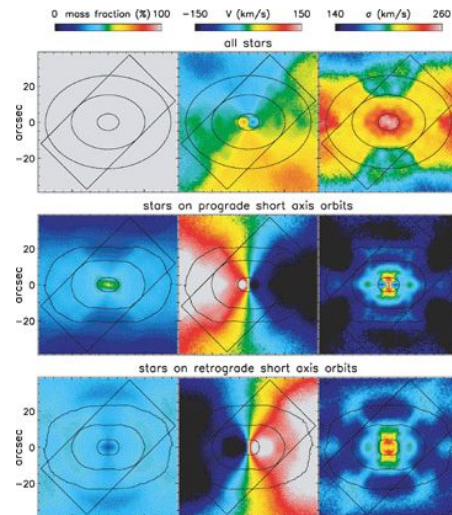
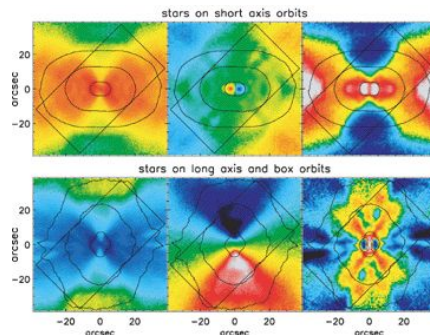
1. Choose a trial potential:

$$\text{Galaxy} = (\text{DM}) + (\text{STARS}) + (\text{BH}) + \dots$$



2. Integrate orbits in a given potential
– store “observations” (i.e., positions, velocities of tracers)

3. Assign weights to orbits →
Reproduce kinematics

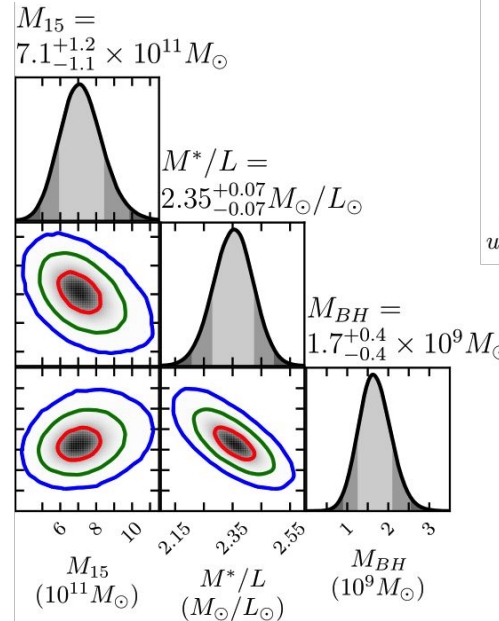


4. Repeat for many potentials
(BH, M/L, DM halo, galaxy shape, etc...) and find best fit to data.

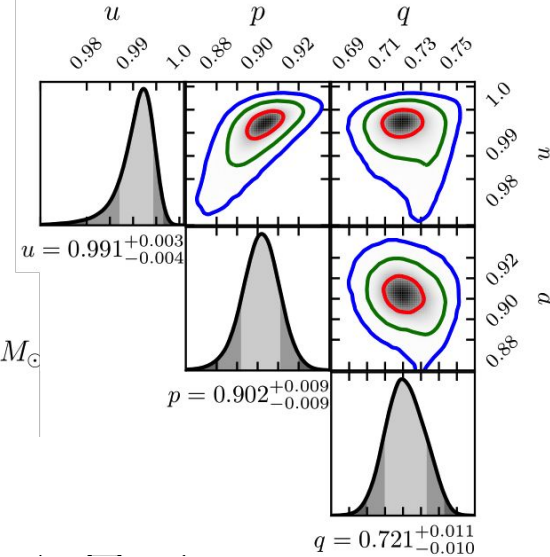
stellar dynamical modeling: applications to NGC 2693

- first simultaneous measurements of DM mass, BH mass, and **intrinsic shapes**
- N2693 has a moderately triaxial intrinsic shape,**
 - Intermediate-to-major axis ratio: **$p = b/a \sim 0.9$**
 - Minor-to-major axis ratio: **$q = c/a \sim 0.7$**
- NGC 2693 $M_{\text{BH}} = (1.7 \pm 0.4) \times 10^9 M_{\text{sun}}$**

mass parameters



shape parameters

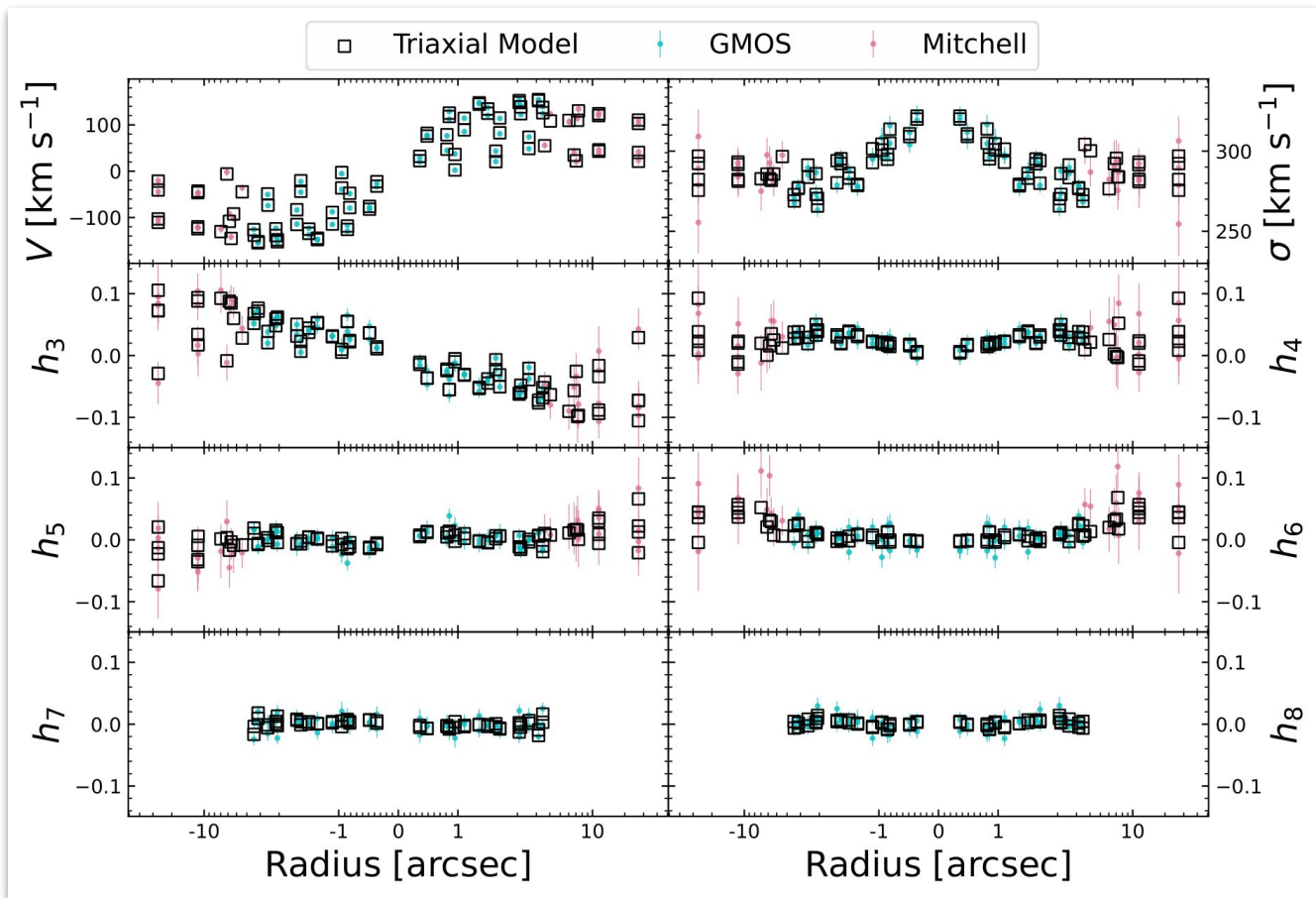


TRIAXIAL

AXISYMMETRIC

NGC 2693 BH: $(1.7 \pm 0.4) \times 10^9 M_{\text{sun}} \rightarrow (2.4 \pm 0.6) \times 10^9 M_{\text{sun}}$

modeling results: applications to NGC 2693

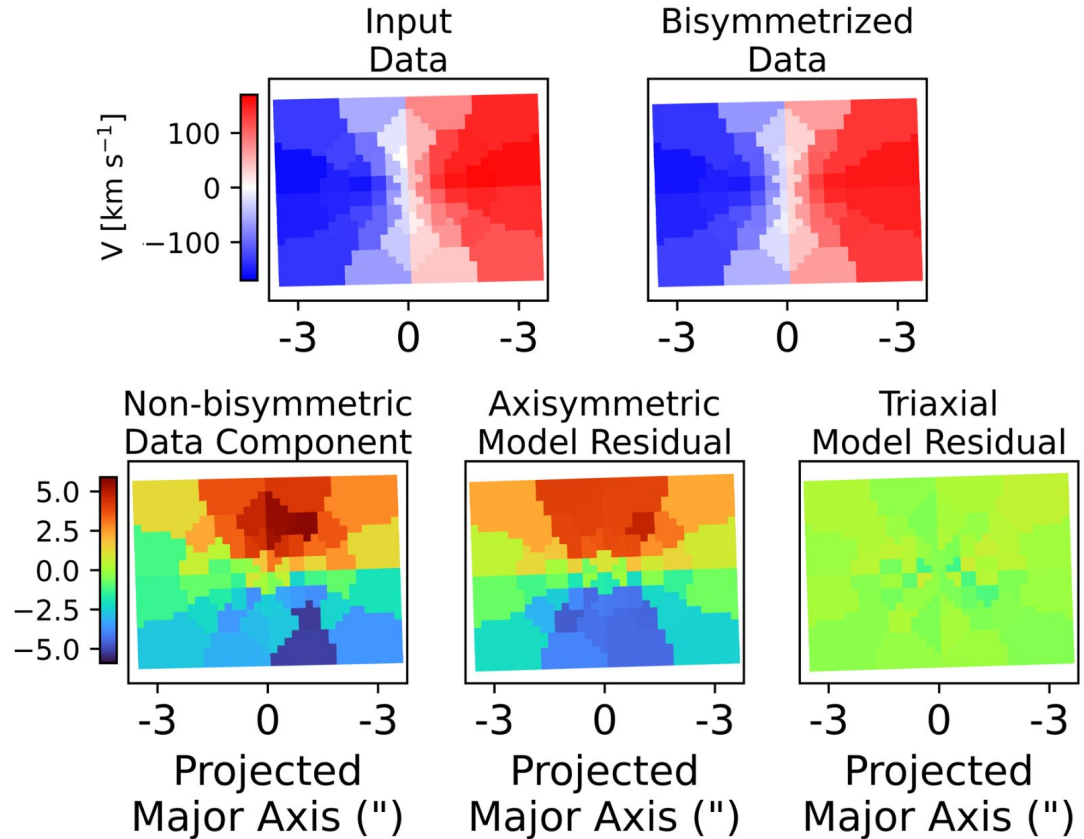


axisymmetric vs. triaxial models

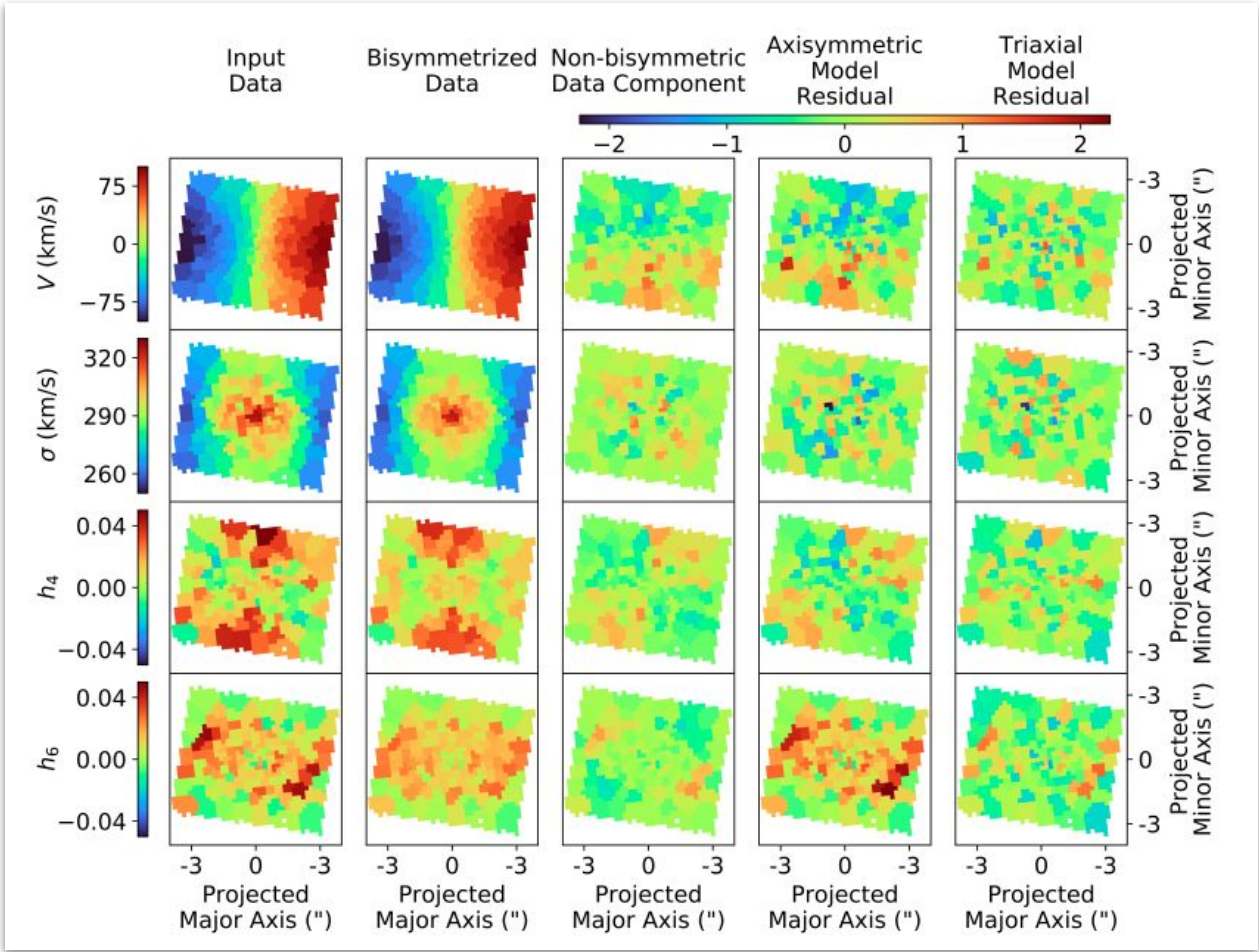
- triaxial model fits data better
- triaxial model also reproduces more complicated velocity structure



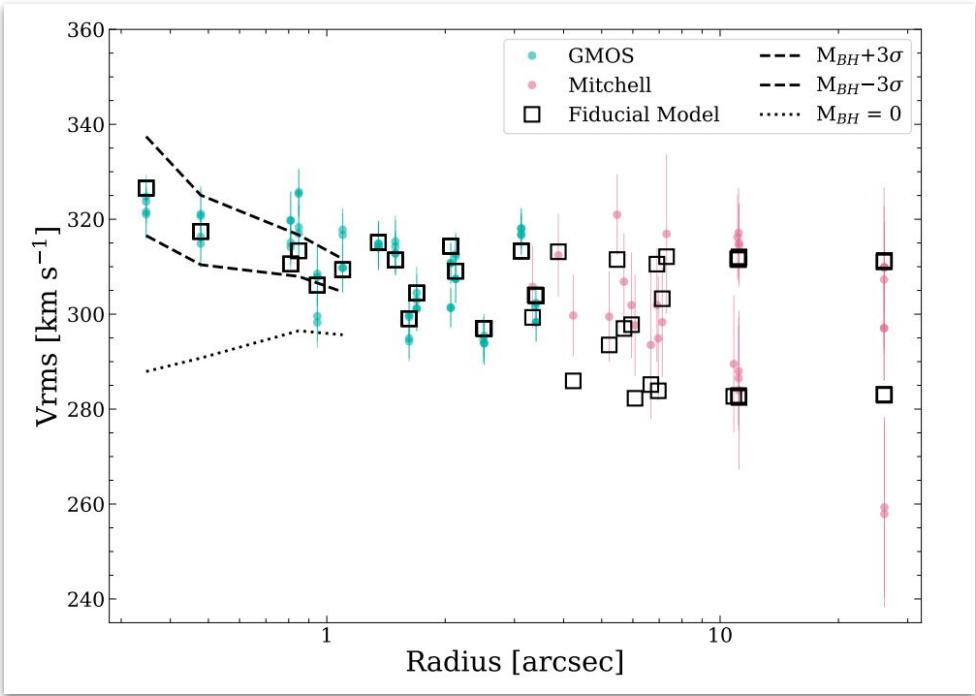
axisymmetric models
cannot reproduce
non-bisymmetric
velocity components



axisymmetric vs. triaxial models



Jeans Modeling



Galaxy Parameter	Triaxial Orbit Model	Axisymmetric Orbit Model	JAM Model
$M_{\text{BH}} [10^9 M_\odot]$	1.7 ± 0.4	2.4 ± 0.6	2.9 ± 0.3
$M^*/L [M_\odot/L_\odot]$	2.35 ± 0.07	2.27 ± 0.1	2.17 ± 0.03
$M_{15} [10^{11} M_\odot]$	$7.1^{+1.2}_{-1.1}$	7.9 ± 1.3	4.7 ± 0.2
β_z	See caption. [†]	See caption. [†]	0.07 ± 0.01
T	0.39 ± 0.04		
T_{maj}	$0.09^{+0.04}_{-0.03}$		
T_{min}	$0.17^{+0.04}_{-0.05}$		
u	$0.991^{+0.003}_{-0.004}$		
p	0.902 ± 0.009		
q	$0.721^{+0.011}_{-0.010}$		
$\theta (^\circ)$	66^{+4}_{-3}		
$\phi (^\circ)$	72 ± 3		
$\psi (^\circ)$	$93.0^{+0.7}_{-0.6}$		

Table 2. Summary of best-fit galaxy models for NGC 2693. For each parameter, we marginalize over the other dimensions and report the 1σ uncertainties. The axisymmetric orbit models and JAM models have fixed inclination of 70° . In orbit models, θ is the inclination angle in the oblate axisymmetric limit ($\psi = 90^\circ$, or equivalently $p = 1$), with $\theta = 90^\circ$ being edge-on and $\theta = 0^\circ$ being face-on. [†]We measure β_z in the orbit model as a function of radius, shown in the bottom panel of Figure 6. The best-fit JAM value of $\beta_z = 0.07 \pm 0.01$ is consistent with the range of β_z values measured from this best-fit model, with values ranging from $\beta_z = -0.27$ at small radii to $\beta_z = 0.28$ at large radii in both the triaxial and axisymmetric Schwarzschild models.

Schwarzschild Modeling Power

our typical scenario:

- (~a few hundred kinematic and astrometric * (~8 Gauss moments per aperture)

~ 10^3 kinematic constraints

typical schwarzschild model:

- $\geq 10^4$ orbits with a free weight parameter

(a) how well can we recover a series of known inputs from our **triaxial** models?

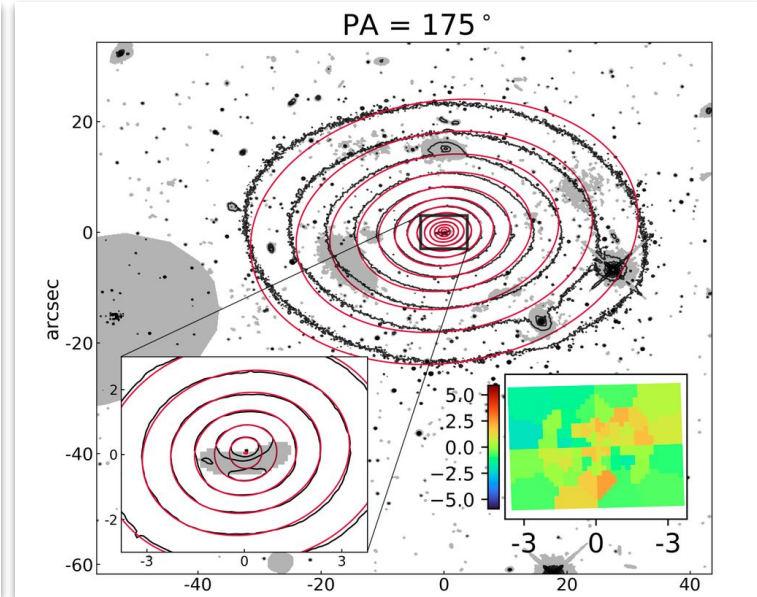
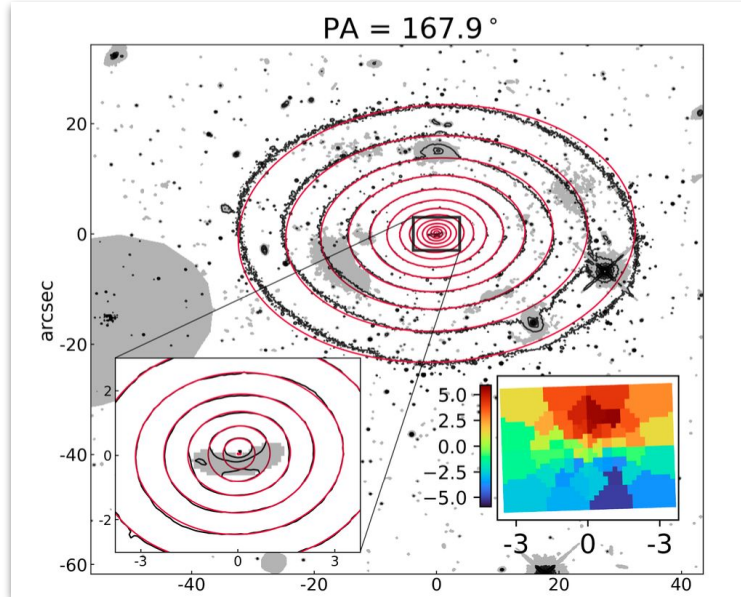
(b) are there analogous biases in our triaxial modeling scheme?

if this were a simple linear model

axisymmetric vs. triaxial models

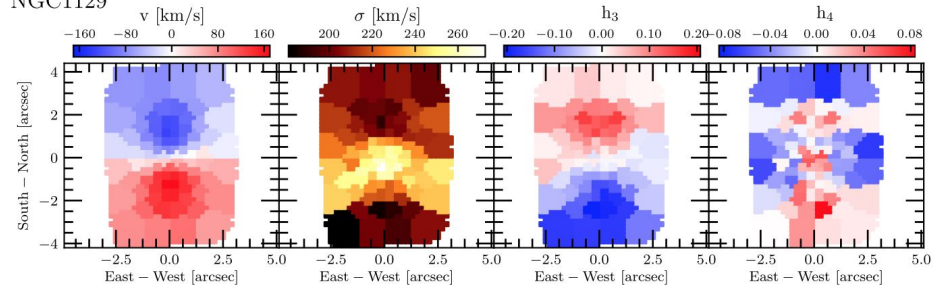


“rotating away” the
non-axisymmetric
component gives an
inconsistent surface
brightness profile

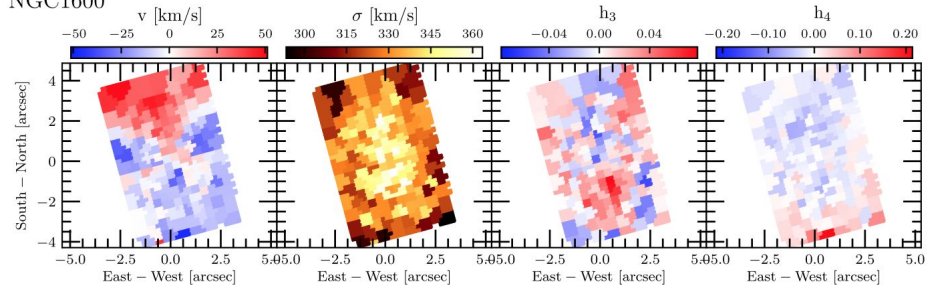


The MASSIVE Survey

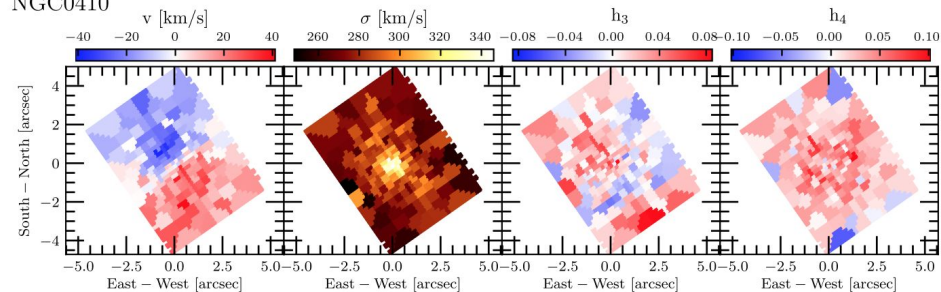
NGC1129



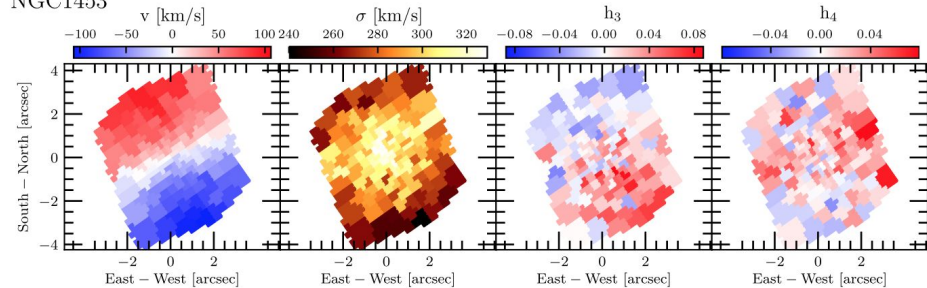
NGC1600



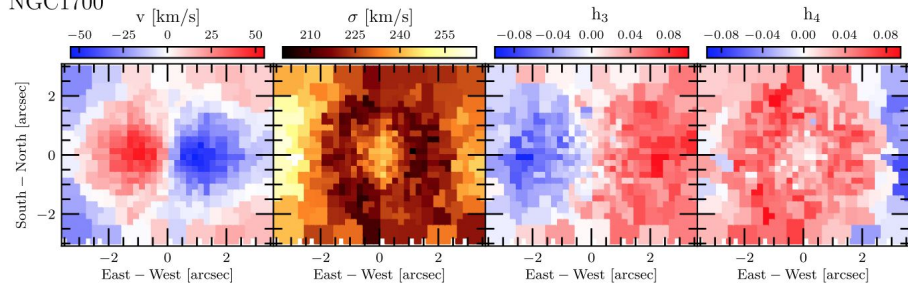
NGC0410



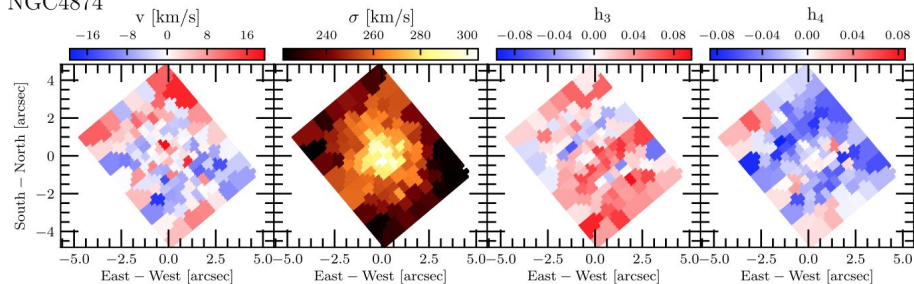
NGC1453



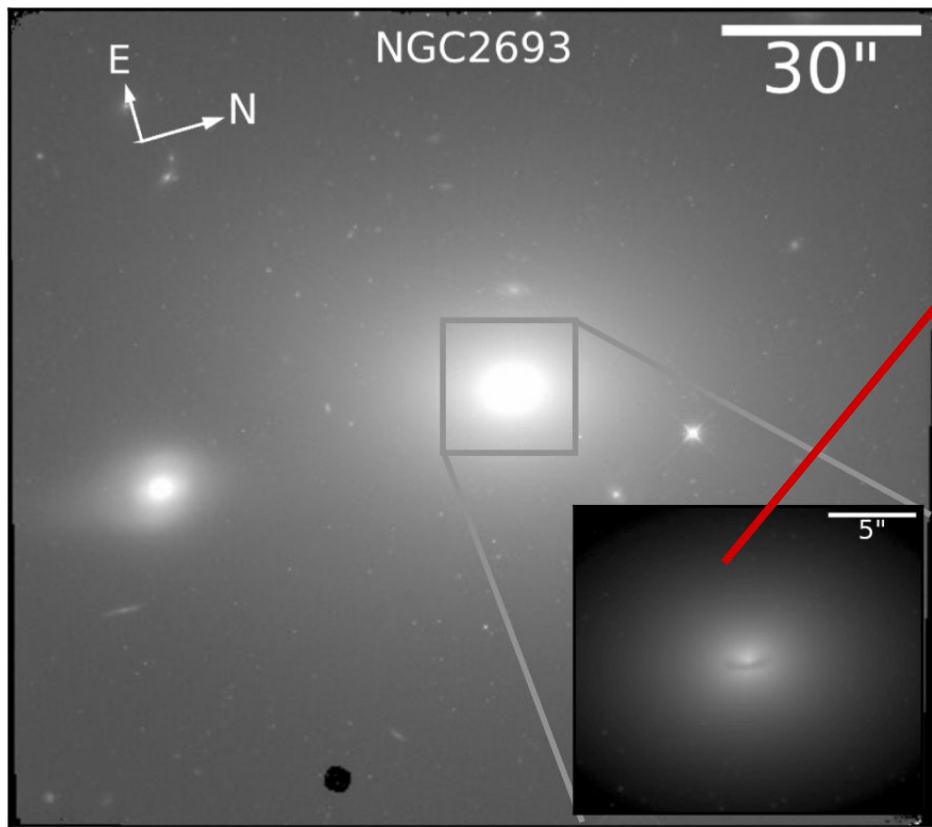
NGC1700



NGC4874



a nice surprise:



**dust disk
inclination:**

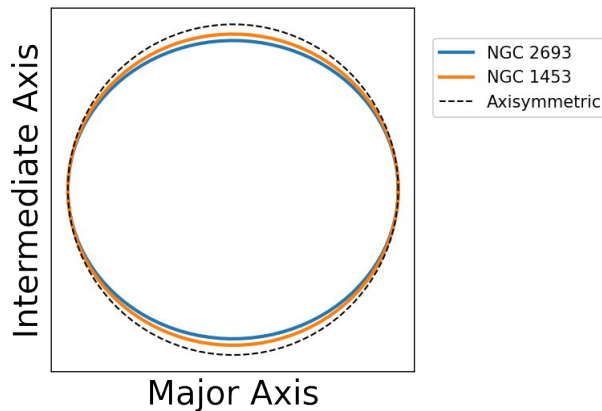
$\theta \sim 70$ degrees

**“inclination”
from the triaxial
model:**

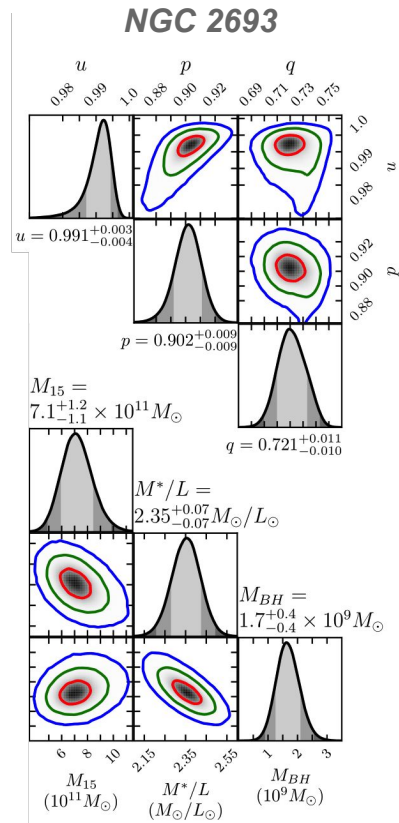
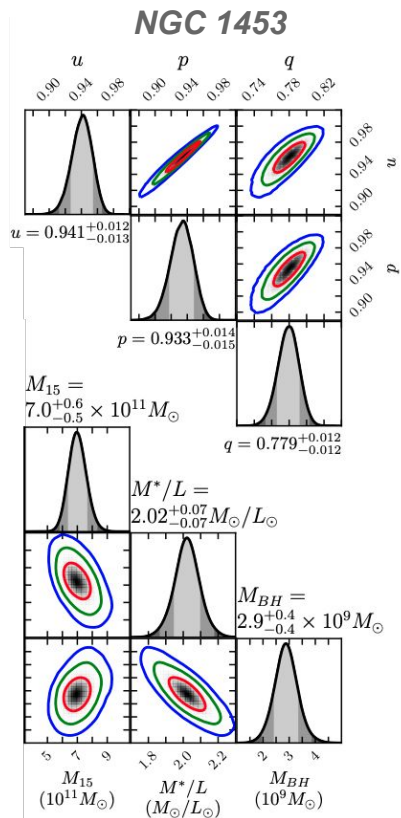
**$\theta = 66^{+4}_{-3}$
degrees**

stellar dynamical modeling: applications to NGC 1453 and 2693

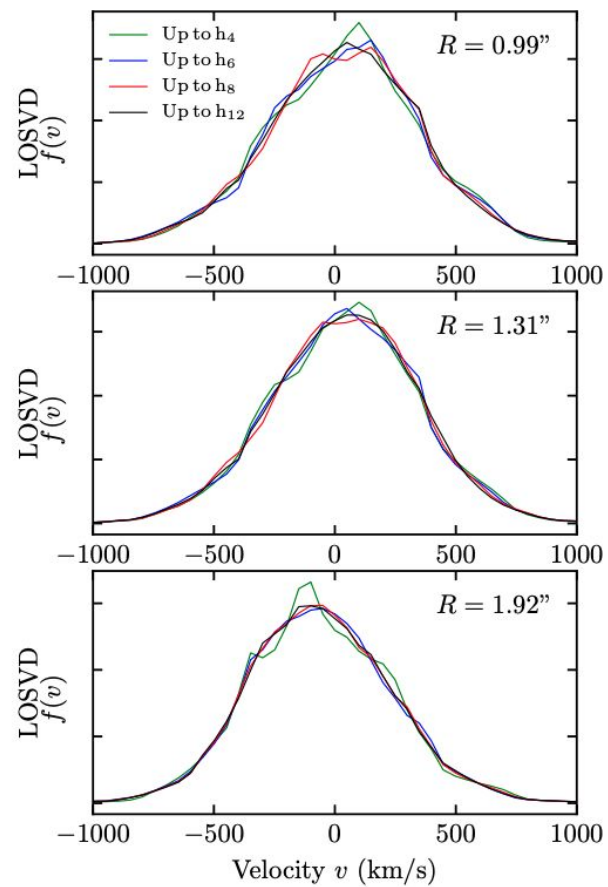
- first simultaneous measurements of DM halo, BH mass, and intrinsic shapes
- neither galaxy is axisymmetric, despite properties suggesting so



- NGC 1453 BH: $2.9 \times 10^9 M_{\text{sun}}$
- NGC 2693 BH: $1.7 \times 10^9 M_{\text{sun}}$



LOSVDs



why **else** are SMBHs important?

1. Estimate BH mass from galaxy properties
where we can't resolve SOI

2. Cross-checking of gas dynamics, mega-maser
disks, and reverberation mapping BH masses

[i.e., Yu+2019 (reverberation mapping), Thater+21 (gas dynamics)]

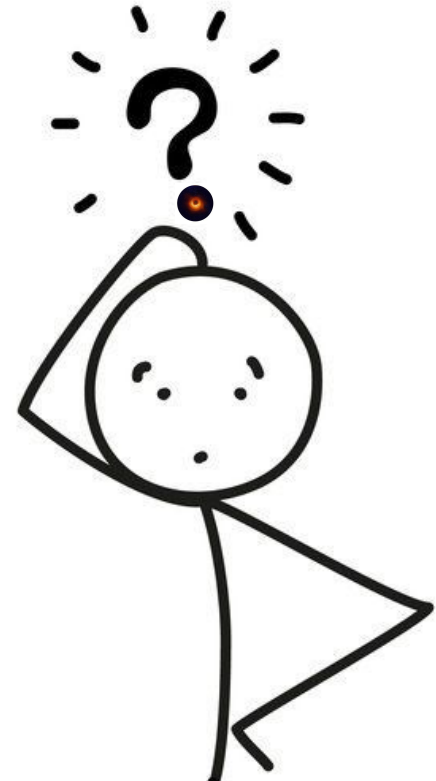
3. $z \sim 0$ mass function/number density \rightarrow predictions
for long- λ gravitational wave signal from Pulsar
Timing Arrays/LISA

[i.e., Shannon+2015, Arzoumanian+2019]

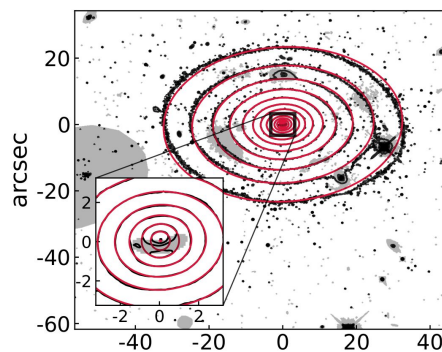
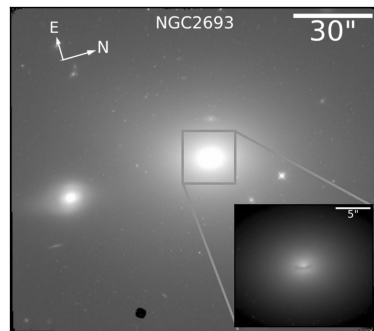
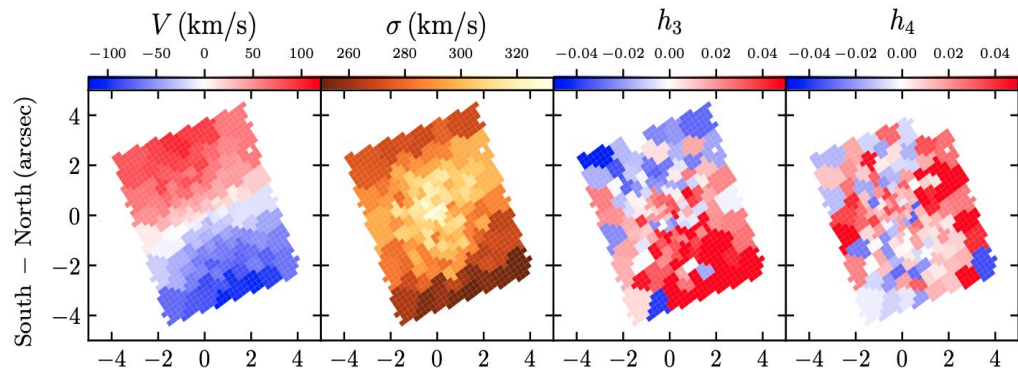
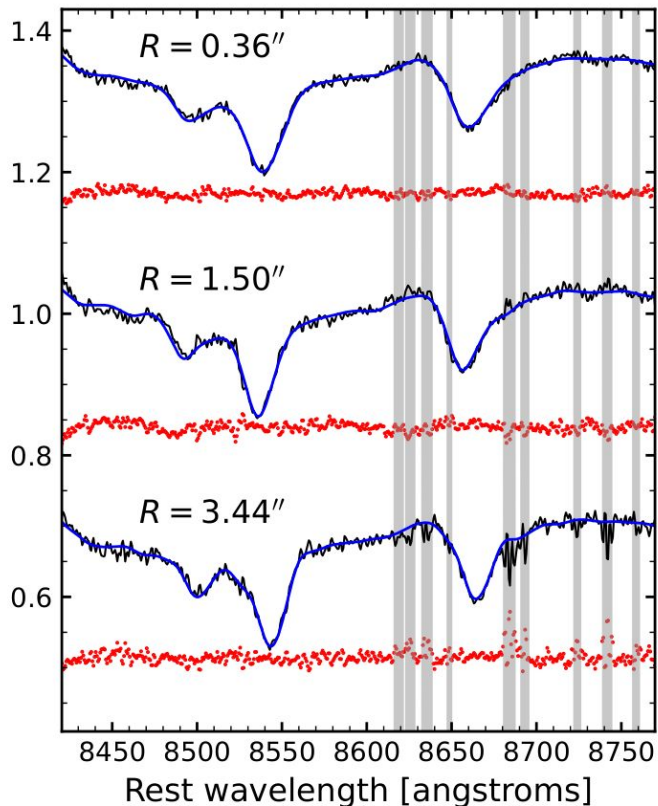
4. Comparison to simulations of AGN feedback
modes and mechanisms

[i.e., Li+2019, Habouzit+2020]

...



stellar dynamical modeling: applications to NGC 1453 and 2693



+ ~10,000 individual
galaxy models

Nested Sampling in A Single Slide

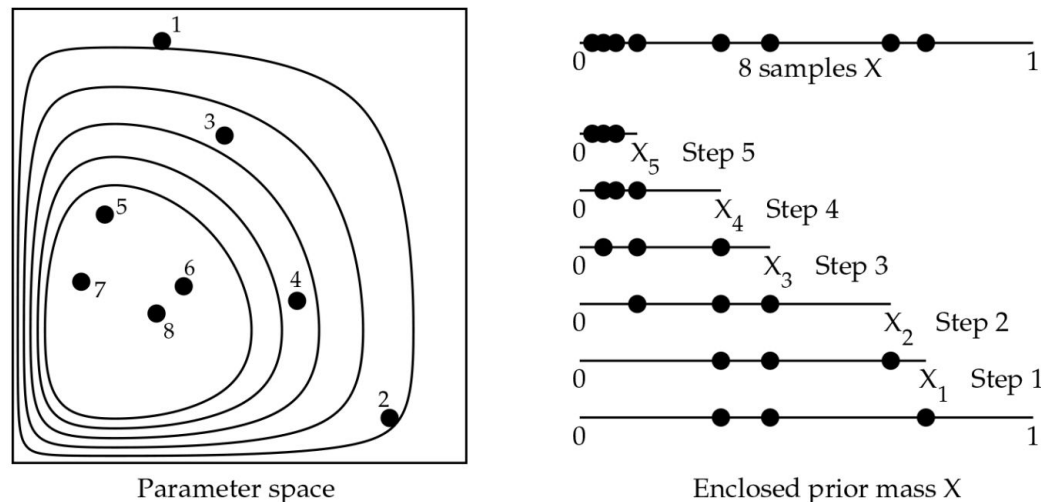


Figure 4: Nested sampling for five steps with a collection of three points. Likelihood contours shrink by factors $\exp(-1/3)$ in area and are roughly followed by successive sample points.

*J. Skilling 2006

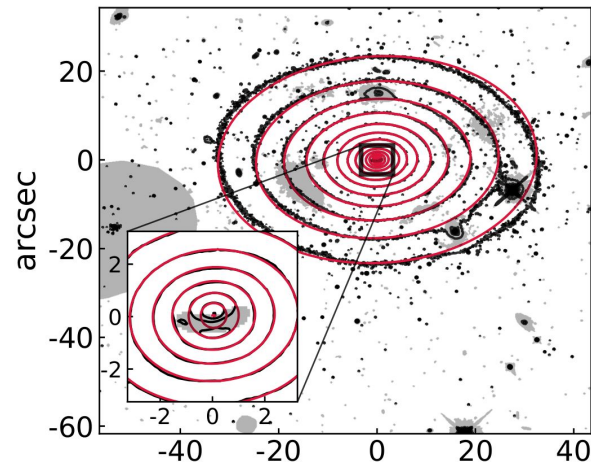
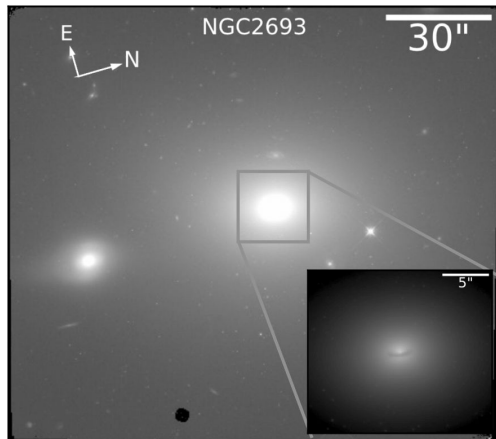
idea : iteratively sample points, getting rid of lowest likelihood at each step → volume shrinks to maxima of distributions

stellar dynamical models

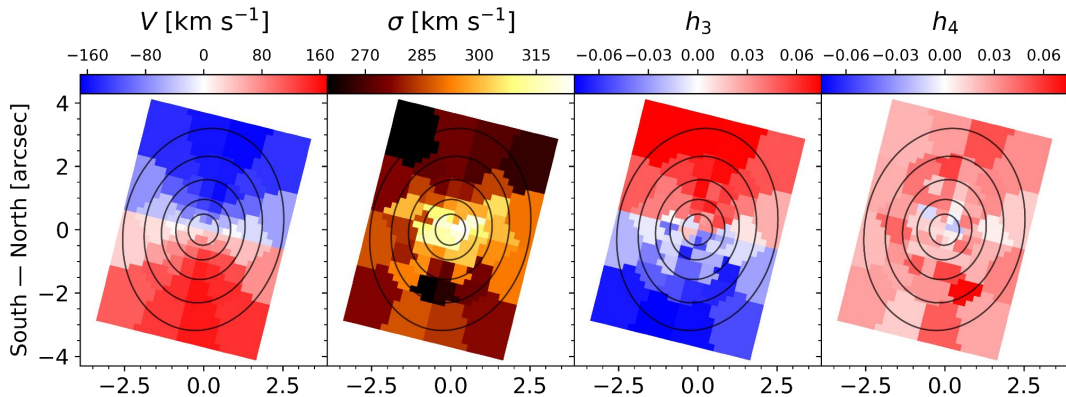
CENTRAL IDEA:

given **kinematics** + **surface brightness** of NGC 2693,
can we determine the
galaxy's mass components
by integrating the orbits of
stars in a gravitational
potential?

surface brightness



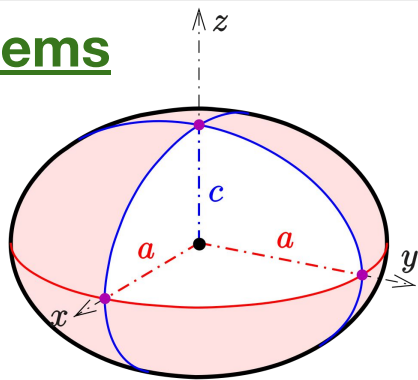
line of sight velocity
distribution



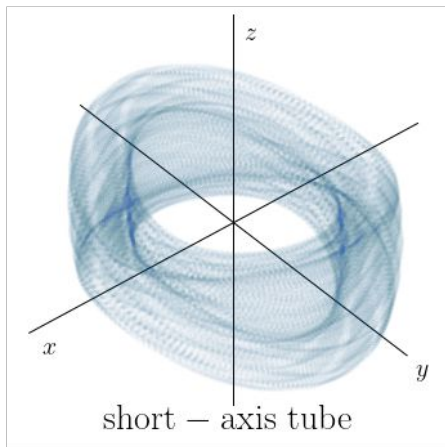
triaxial dynamical modeling

axisymmetric systems

- 4 parameters:
 - BH
 - M/L
 - DM Halo
 - Inclination

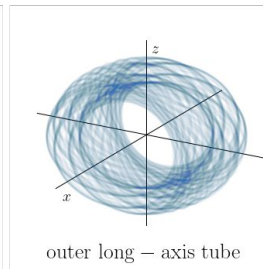
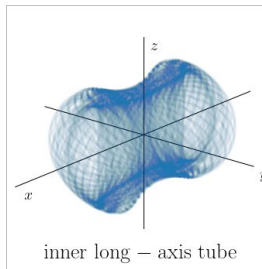
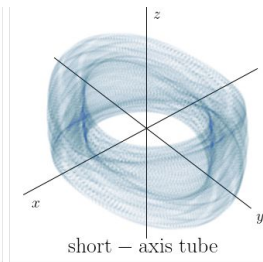
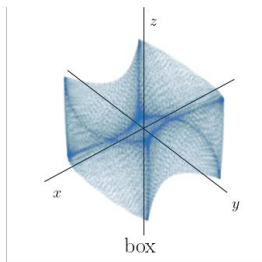
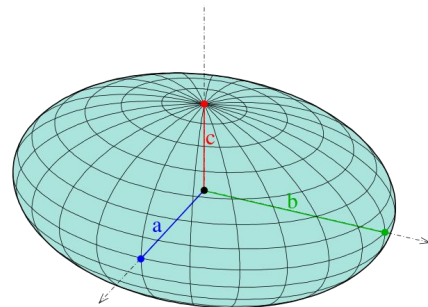


important:
axisymmetric
systems are, by
construction,
bisymmetric



triaxial systems

- 6 parameters:
 - BH
 - M/L
 - DM Halo
 - 3 Shape Parameters

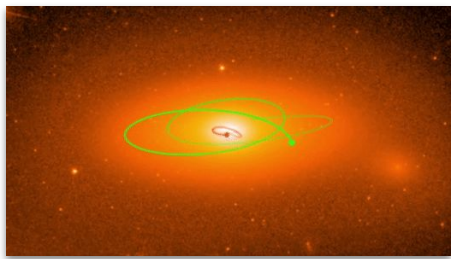


triaxial dynamical modeling

1. Choose a trial potential:

$$\text{Galaxy} = (\text{DM}) + (\text{STARS}) + (\text{BH}) + \dots$$

2. Generate stellar orbits in trial potential

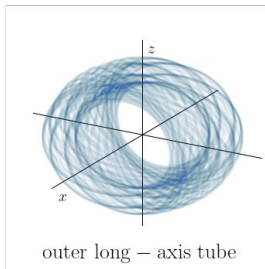
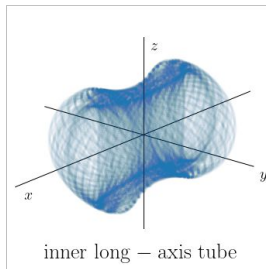
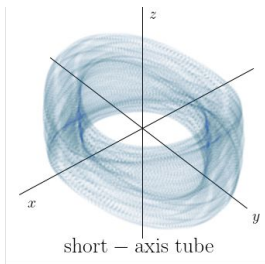
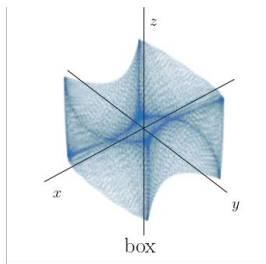
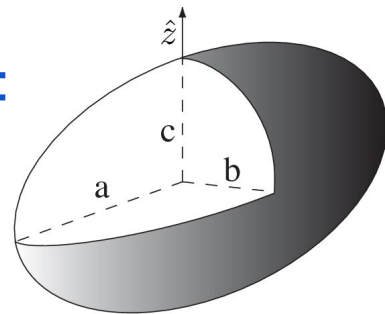


3. Determine which orbits most accurately reproduce **kinematics** + **photometry** for a *single* trial potential

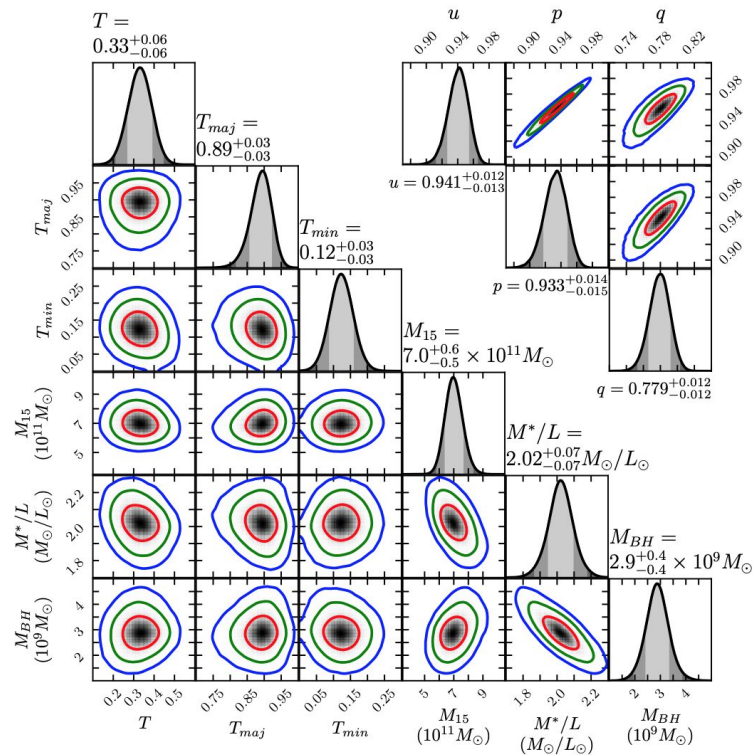
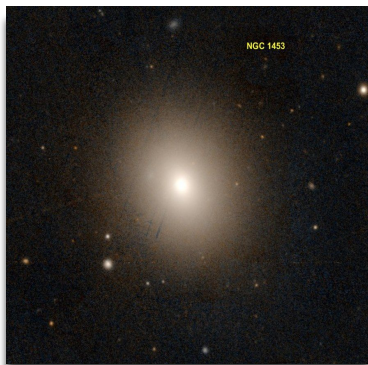
4. Find which assumed potential fits **kinematics** + **photometry** best **across** trial potentials (BH, ML, Shapes)

triaxial systems

- 6 knobs to turn:
 - BH
 - M/L
 - DM Halo
 - 3 Shape Parameters

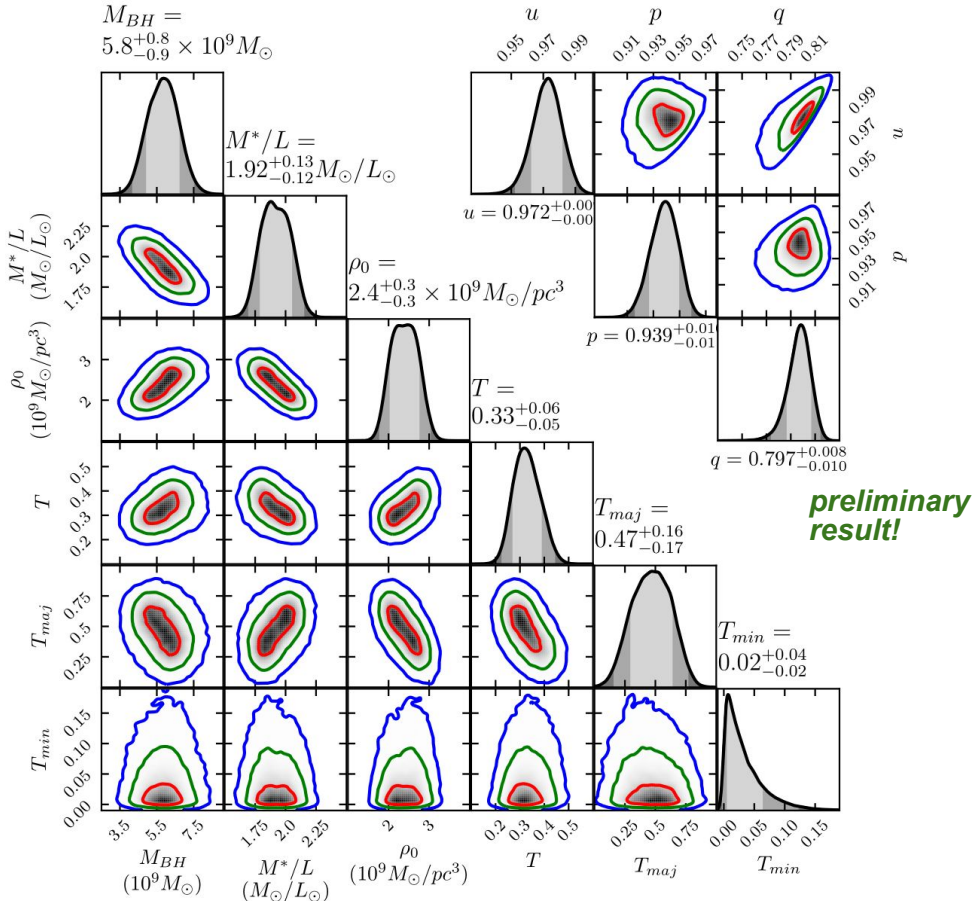
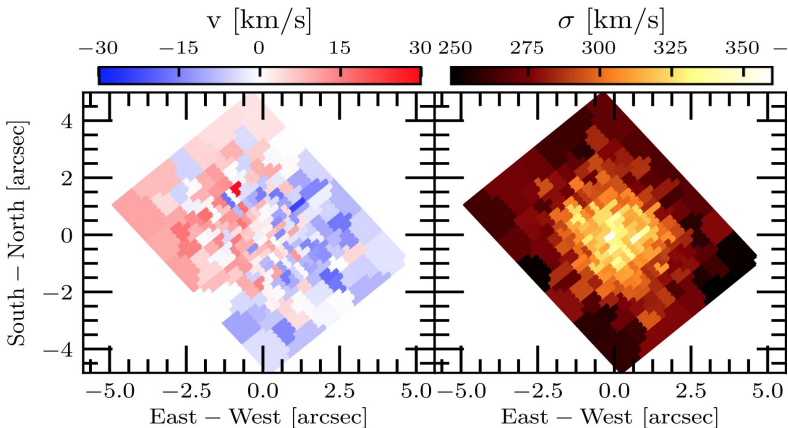


stellar dynamical modeling: other applied cases



stellar dynamical modeling: ongoing work

NGC 57

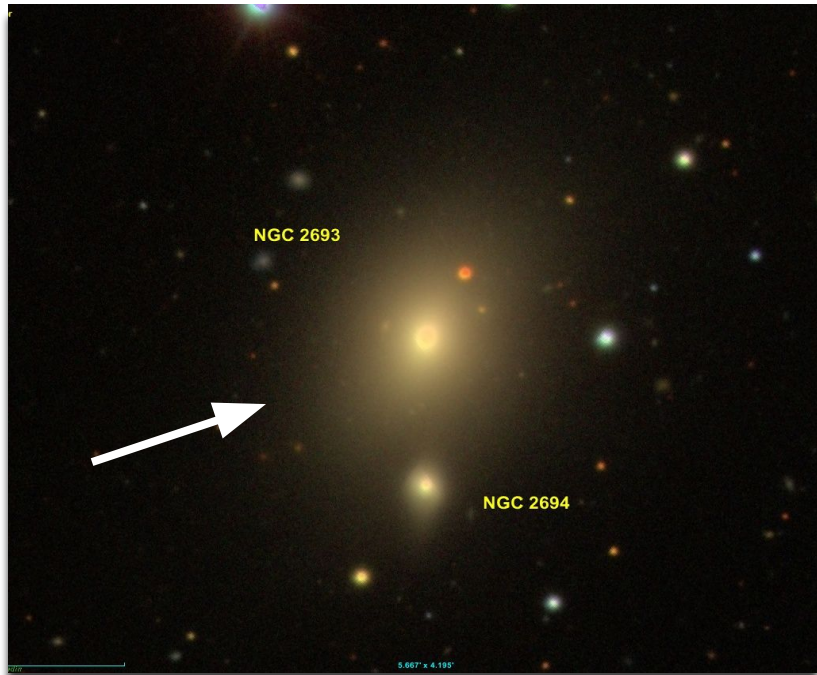
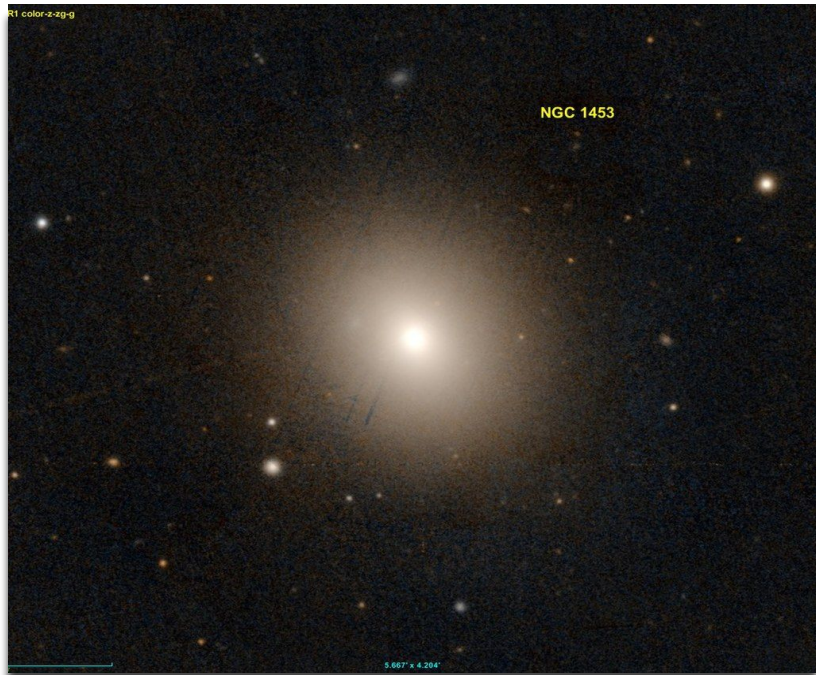


preliminary
result!

Triaxial Result: $M_{BH} \sim 6 \times 10^9 M_{sun}$

- Another simultaneous measurement of mass components + intrinsic shape of a massive elliptical

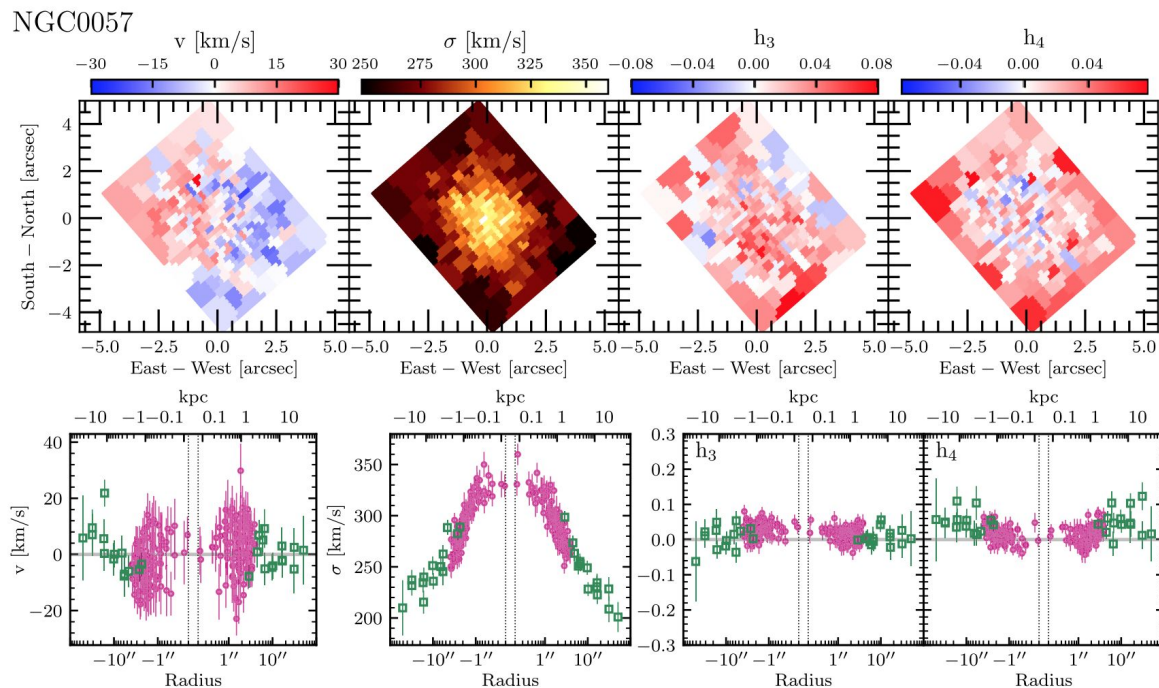
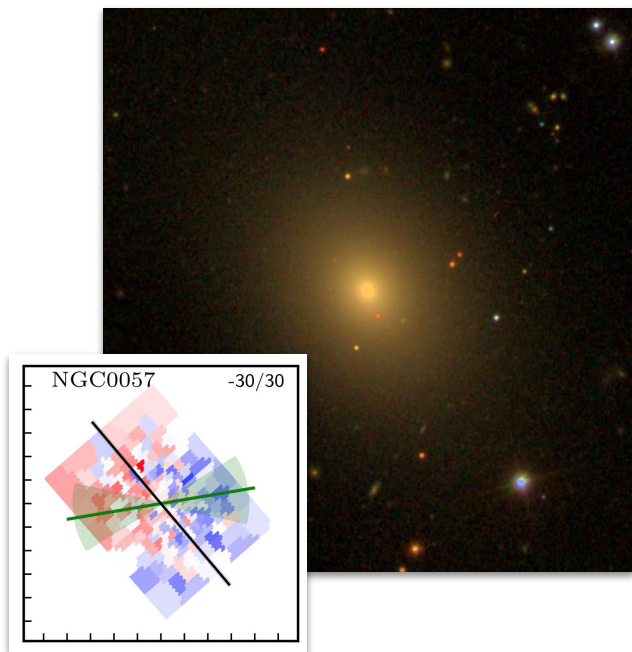
stellar kinematics of NGC 1453 and 2693



SDSS DR9 Images

stellar dynamical modeling: ongoing work

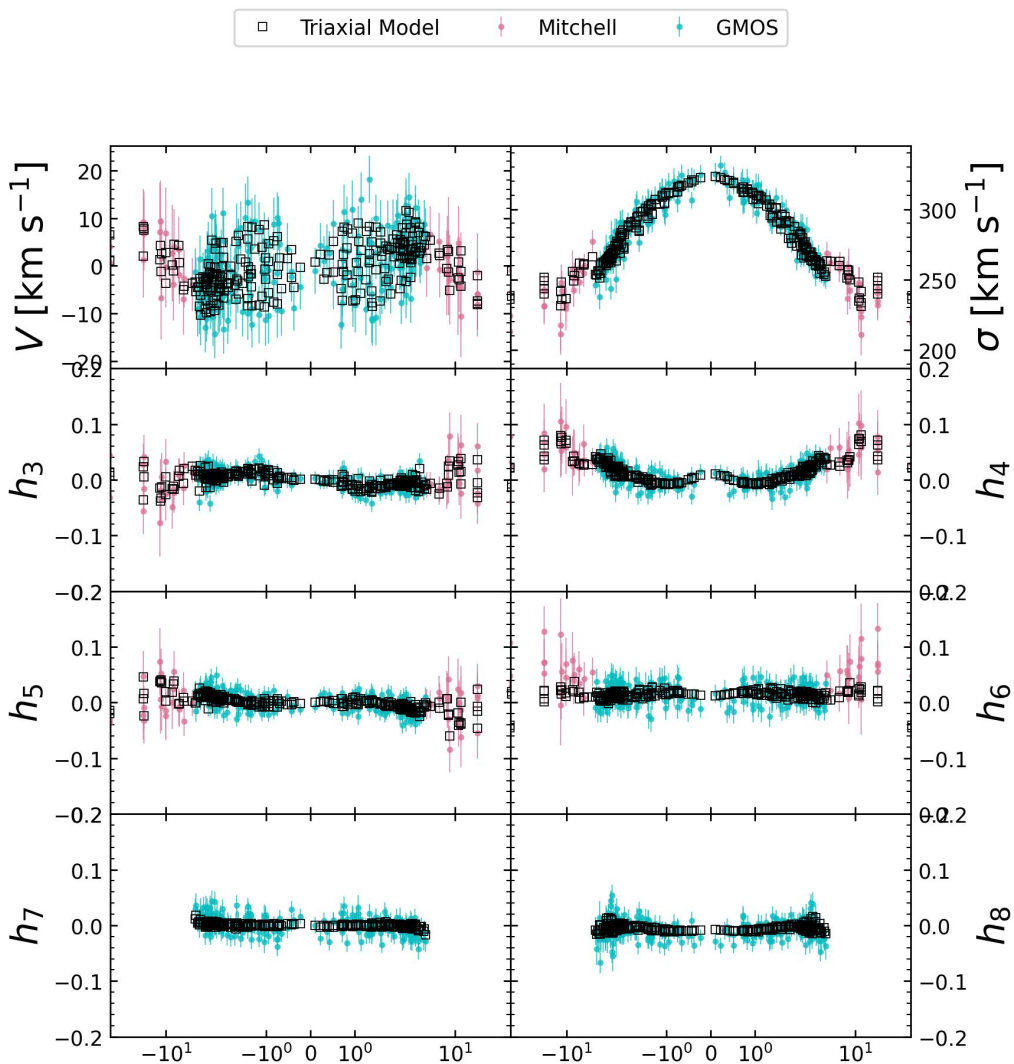
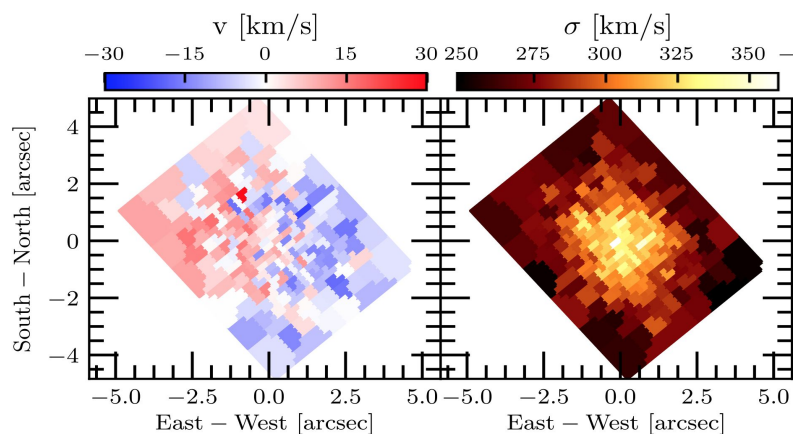
NGC 57



- Slow rotation (~ 25 km/s),
kinematically and
photometrically aligned

stellar dynamical modeling: ongoing work

NGC 57

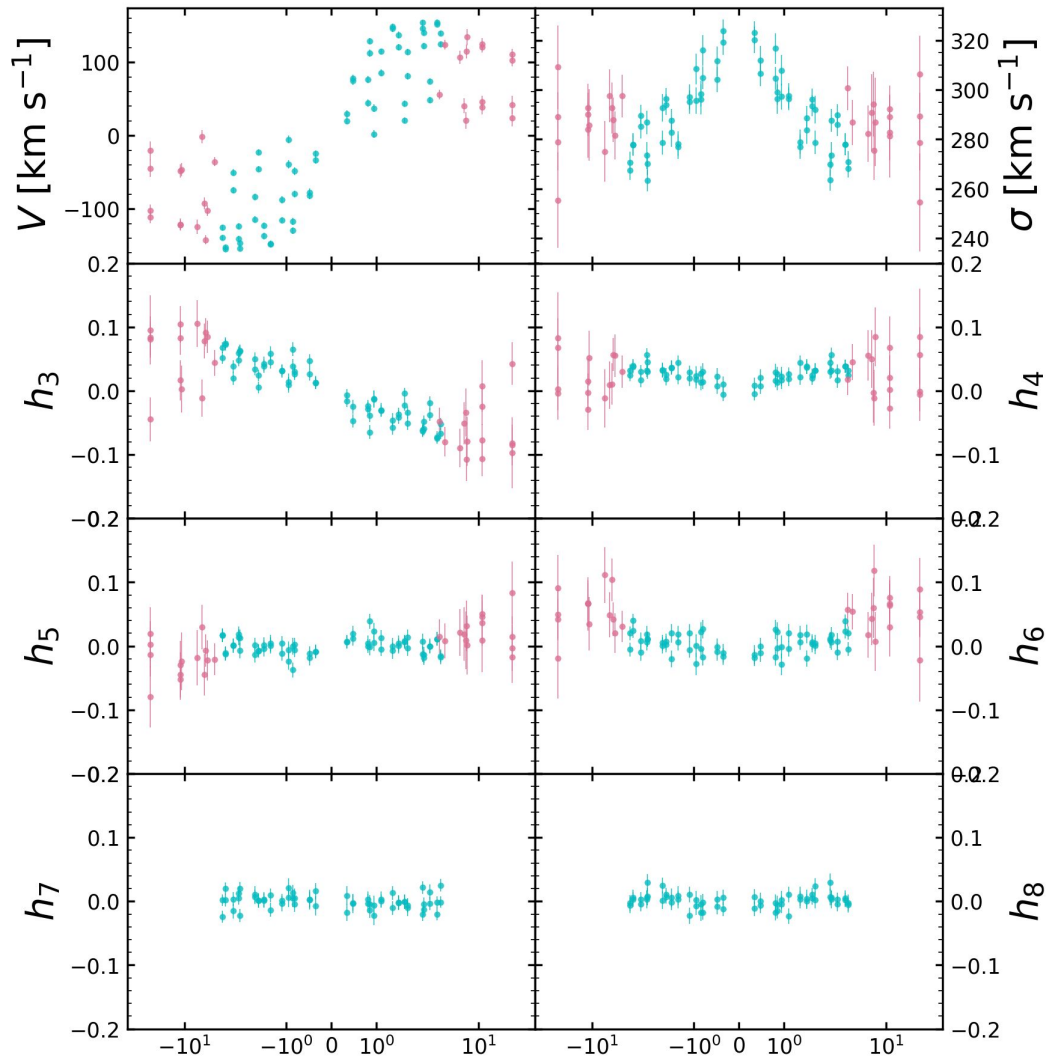


Transition to Mock Results

- Mention contrasting axisymmetric M_{bh} /difference with triaxial might be because of modeling differences, with inclination as the example case of parameter bias – could this be the underlying difference between recovered mass parameters?
 - Need to be a bit more elaborative on axisymmetric modeling if this is the case. Currently only describe triaxial modeling
- Motivate this with an example of fits with different numbers of d.o.f to show the difference (something simple), and a primer on m_{eff}
- Our machinery seems to reliably recover the inputs, and that our parameter recovery are not strongly dependent on the degrees of freedom in our models
- Additionally have started to investigate more interpretable measurements of model flexibility for schwarzschild models,

Schwarzschild Model Flexibility: Axisymmetric Models

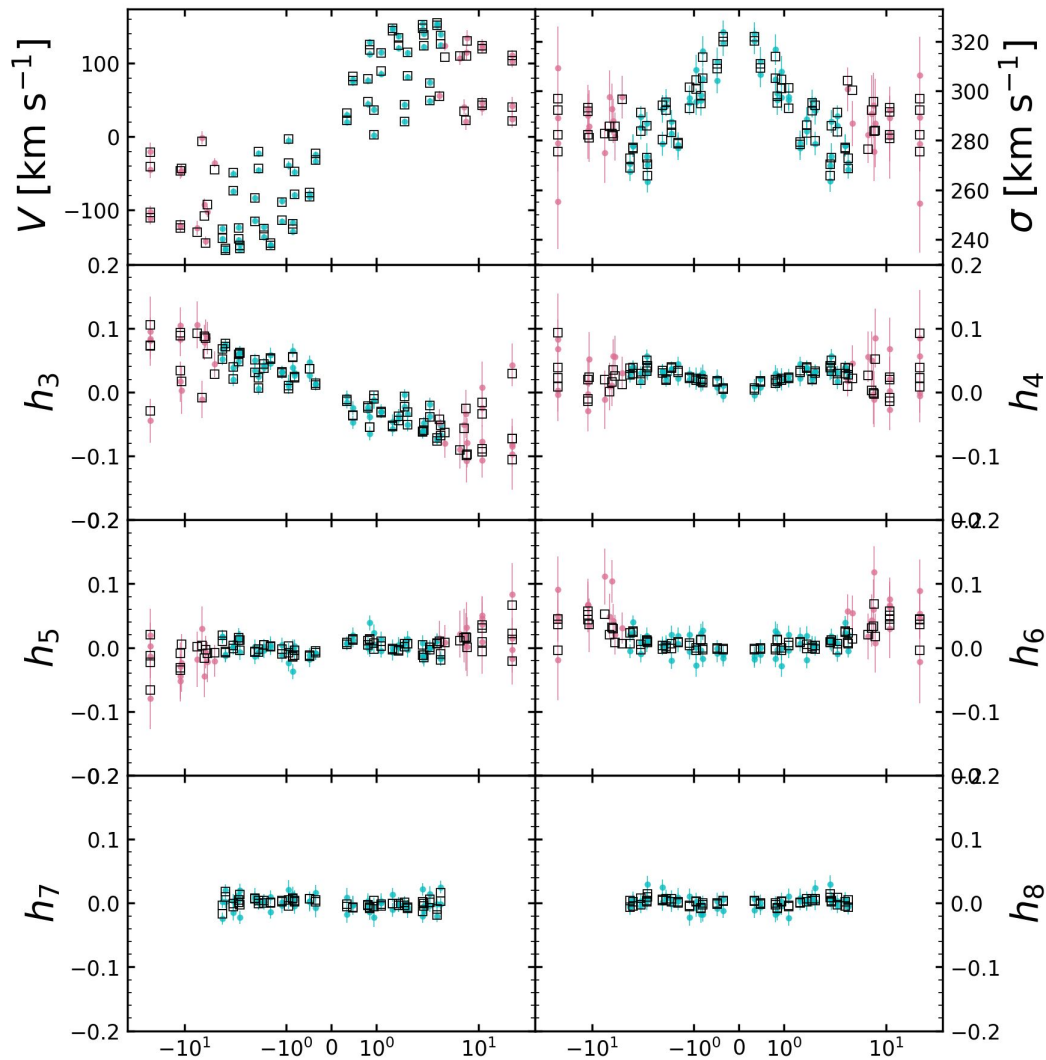
consider the following
scenario:



Schwarzschild Model Flexibility: Axisymmetric Models

consider the following
scenario:

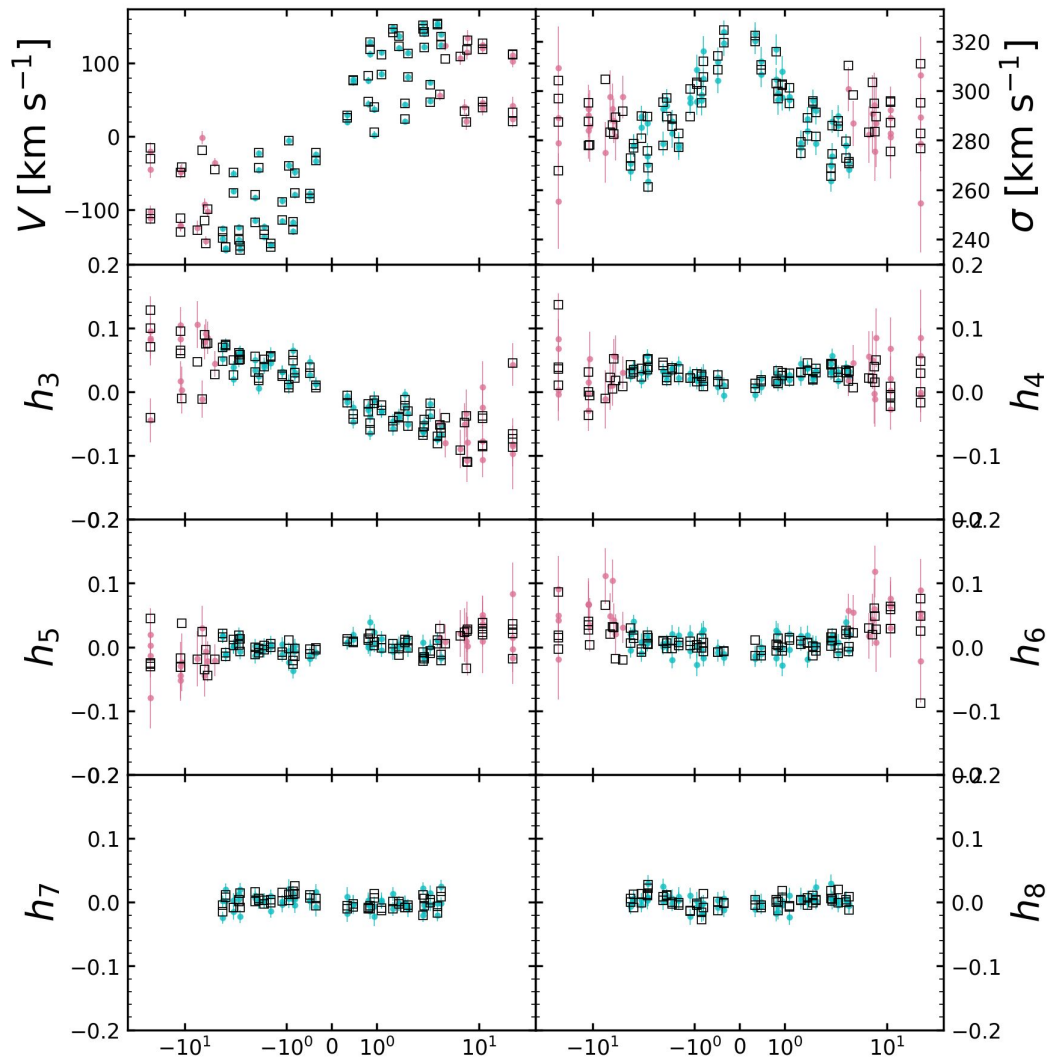
- We fit a Schwarzschild model to NGC 2693's data with the measured M_{BH} , M/L , DM halo, and $i = 70^\circ$



Schwarzschild Model Flexibility: Axisymmetric Models

consider the following scenario:

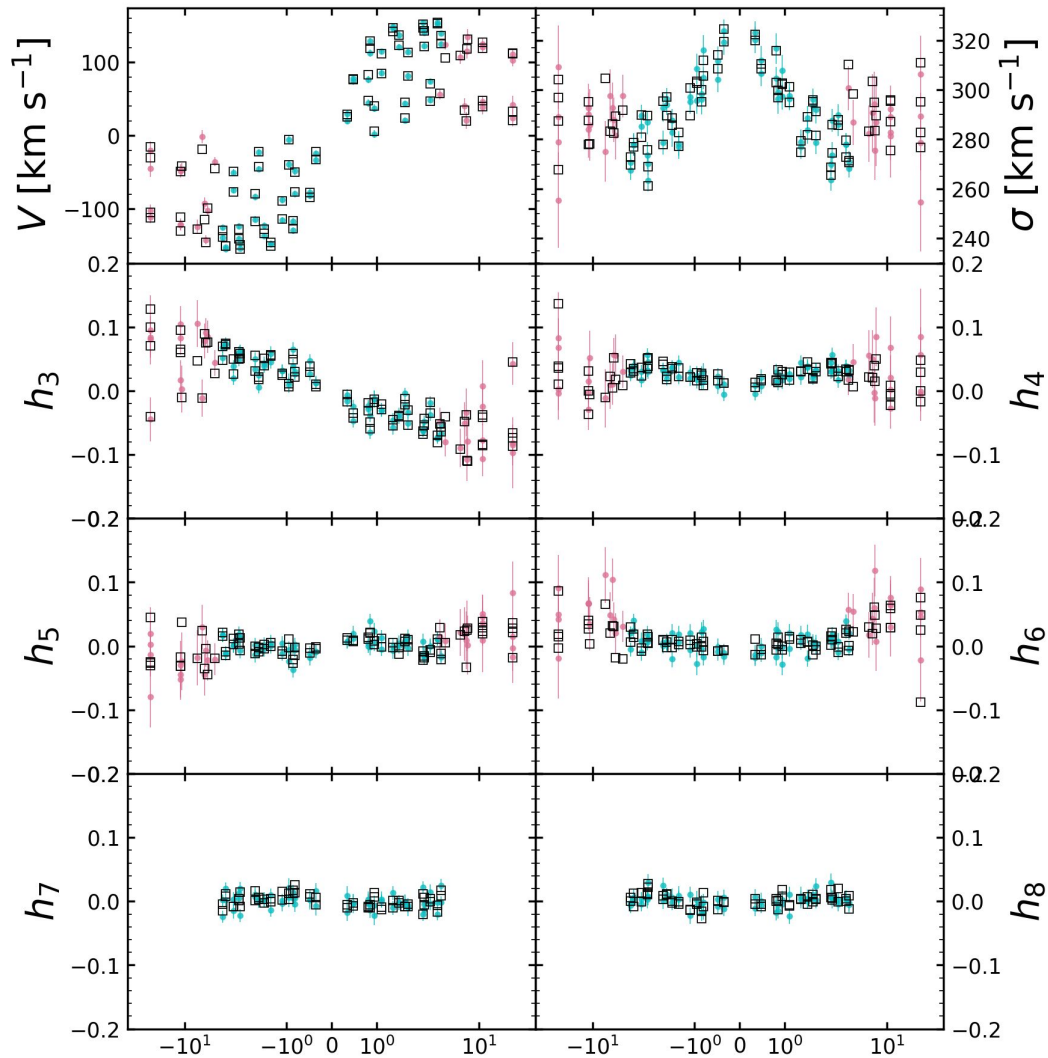
- We fit a Schwarzschild model to NGC 2693's data with the measured M_{BH} , M/L , DM halo, and $i = 70^\circ$
- Perturb the model fit by the measurement uncertainty



Schwarzschild Model Flexibility: Axisymmetric Models

consider the following scenario:

- We fit a Schwarzschild model to NGC 2693's data with the measured M_{BH} , M/L , DM halo, and $i = 70^\circ$
- Perturb the model fit by the measurement uncertainty
- Feed this as “input” to our code architecture and try to recover the correct inputs



Schwarzschild Modeling Power

our typical scenario:

- (~a few hundred kinematic apertures)
* (~8 Gauss-Hermite moments per aperture)

~ 10^3 kinematic constraints

typical schwarzschild model:

- $\geq 10^4$ orbits with a free weight parameter

$$\chi_k^2 \rightarrow 0$$

k (degrees of freedom)
= (# data points) - (# params)
= **negative**

if this were a simple linear model

Schwarzschild Model Flexibility (pt 2)

motivating why we might expect this in axisymmetric models:

- both prograde and retrograde orbits are included in our orbits libraries, which have opposite signs and contribute maximally to the LOSVD in edge-on orientations; this in turn gives a higher effective degrees of freedom in these high i models
- In other words, they're more **flexible** as you move toward $i \rightarrow 90^\circ$
- Solution: apply a penalty which quantifies the flexibility of these models (have a slide motivating a penalty like $m_{\text{eff}}/\text{Norb}$, show results of penalizing the models as such)

Schwarzschild Model Flexibility (pt 3)

Triaxial models and current work:

- Currently evaluating if there is a similar bias in triaxial models which don't have the same symmetries in the orbits, plus have different families of orbits entirely
- Initial results seems to suggest that we don't need a penalty term for our triaxial models, and that we can accurately recover the parameters associated with a set of input kinematics